Amplitude Clipping and Iterative Reconstruction of MIMO-OFDM Signals with Optimum Equalization

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Abstract—Multi-input multi-output orthogonal frequency-division multiplexing (MIMO-OFDM) has become a promising candidate for next generation broadband wireless communications. However, like a single-input single-output (SISO)-OFDM, one main disadvantage of the MIMO-OFDM is the high peak-to-average power ratio (PAPR), which can be reduced by using an amplitude clipping. In this paper, we propose clipped signal reconstruction methods for the MIMO-OFDM with spatial diversity, such as space-time and space-frequency block codes (STBC/SFBC). The proposed methods are based on the technique called iterative amplitude reconstruction (IAR) for SISO-OFDM. It is shown that the IAR can be easily employed for the STBC-OFDM, but it cannot be directly applied to the SFBC-OFDM, because the transmitted sequences over different antennas are dependent due to the use of space-frequency code. We propose a new SFBC transmitter for clipped OFDM, which has approximately half the computational complexity of conventional SFBC-OFDM. The proposed clipping preserves the orthogonality of transmitted signals, and the clipped signals are iteratively recovered at the receiver. Further, we theoretically analyze the performance of IAR with optimum equalization, and also provide highly accurate channel estimation of the OFDM with amplitude clipping. Simulation results show that the proposed receivers effectively recover contaminated OFDM signals with a moderate computational complexity.

Index Terms—Orthogonal frequency-division multiplexing (OFDM), amplitude clipping, space-time and space-frequency block code (STBC/SFBC), iterative amplitude reconstruction (IAR).

I. INTRODUCTION

In various fora and organizations for wireless research, such as the Wireless World Research Forum (WWRF) and International Telecommunication Union (ITU), there have been active discussions about 4G or beyond 3G systems to be deployed around 2010 [1], [2]. Multi-input multi-output orthogonal frequency-division multiplexing (MIMO-OFDM) has drawn significant interests as a candidate for the 4G or beyond 3G modulation techniques, due to the high bandwidth efficiency and the robustness against multipath fading channels [4]-[6]. However, MIMO-OFDM, like a single-input single-output (SISO)-OFDM, has an inherent drawback of high peak-to-average power ratio (PAPR). High PAPR requires large dynamic range of the transmit power amplifier and reduces the power efficiency, and thus the cost of transmitter is increased and the battery life time is decreased.

A number of approaches have been proposed to cope with the SISO-OFDM PAPR problem. First, techniques based on the channel coding transmit only the codewords with low PAPR [8], [9]. Such coding techniques offer good PAPR reduction and coding gain. The critical problem of coding approach is that for an OFDM system with large number of subcarriers, either it encounters design difficulties or the coding rate becomes prohibitively low. Phase rotation is another approach to reduce PAPR, including selective mapping (SLM) [10], and partial transmit sequence (PTS) [11]. It generates a set of sufficiently different candidate data blocks, all representing the same information as the original data block, and selects the most favorable block for transmission. Although the phase rotation works with an arbitrary number of subcarriers and any modulation schemes, it requires high computational complexity and side information. Deliberate amplitude clipping [12]-[17] may be one of the most effective solutions when the number of subcarriers is large. It clearly removes the amplitude peak, and does not introduce redundancy and power increase. Clipping, however, causes distortion that degrades the system performance.

To mitigate the harmful effects of the clipping, a few techniques have been proposed. In [19], a scheme called decision-aided reconstruction (DAR) is proposed, and oversampled signals are used to compensate for the SNR degradation due to the clipping [20]. However, the techniques in [19] and [20] work well at high clipping ratio (CR) values, and the technique in [20] needs bandwidth expansions. In [21], [22], maximum-likelihood detection methods are proposed for clipped OFDM signals. In this paper, based on the DAR, we have proposed a technique called iterative amplitude reconstruction (IAR) for coded OFDM, which recovers clipped signals by comparing the estimates of clipped and nonclipped OFDM samples [18]. Since the IAR is an iterative technique, the reliability of initial estimates and the propagation error to the next iteration considerably affect the overall system performance. In this paper, we theoretically show that the optimum equalization of IAR increases the reliability of initial estimates, and the power of propagation error to the next iteration is approximately half of the DAR. Further, we first propose clipped signal reconstruction methods for MIMO-OFDMs with spatial diversity, such as space-time and space-frequency block codes (STBC/SFBC). It is shown that the IAR can be easily employed for the STBC-OFDM. However, unlike the STBC-OFDM, where independent OFDM sequences are transmitted over different antennas at certain time, the IAR cannot be directly applied to the SFBC-OFDM, because the sequences over different antennas in SFBC-OFDM are dependent due to...
the use of space-frequency code. In this paper, we propose a new SFBC transmitter for the clipped OFDM, which has approximately half the computational complexity of conventional SFBC-OFDM. The proposed clipping preserves the orthogonality of transmitted signals, and the clipped signals are iteratively recovered at the receiver based on the IAR. We also provide a channel estimation of the clipped OFDM by using a sequence, which has constant amplitude in both frequency and time domains. Simulation results are presented to verify our analysis and to provide further insights into the performance.

The remainder of this paper is organized as follows. In Section II, the system model of clipped OFDM is described, and the technique called IAR is presented and analyzed in Section III. In Section IV, we extend the IAR into the STBC/SFBC MIMO-OFDMs. A new SFBC transmitter for clipped OFDM and its iterative signal reconstruction method are proposed in Section V. In Section VI, channel estimation and simulation results of the proposed techniques are presented, and the paper is concluded in Section VII.

II. SYSTEM MODEL OF CLIPPED OFDM

The complex baseband OFDM signal with N subcarriers is expressed as

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_m[n] \exp\{j2\pi nf_0t\}, \quad 0 \leq t \leq T \quad (1) \]

where \( f_0 \) is the subcarrier spacing, \( T \) is the OFDM block duration, and \( \{X_m[n]\}_{n=0}^{N-1} \) is the symbol sequence to be transmitted for the \( m \)th OFDM block. The discrete time OFDM signals, \( \{x_m[k]\}_{k=0}^{JN-1} \), sampled at the time interval \( \Delta t = T/JN \), can be obtained by padding \( \{X_m[n]\}_{n=0}^{N-1} \) with \((J-1)N\) zeros and taking \( JN\)-point inverse fast Fourier transform (IFFT). The parameter \( J \) is an oversampling factor, and \( J = 1 \) is for the Nyquist rate sampling. Clipping is performed on the IFFT output sequence as

\[ \bar{x}_m[k] = \begin{cases} x_m[k], & |x_m[k]| \leq A, \\ A \exp\{\arg(x_m[k])\}, & |x_m[k]| > A, \\ \end{cases} \quad 0 \leq k \leq JN - 1 \quad (2) \]

where \( \bar{x}_m[k] \) is the clipped sample and \( A \) is the clipping amplitude. If the clipping is performed on the Nyquist sampling rate, it causes clipping noise to fall all in-band and suffers considerable peak regrowth after the D/A conversion. When the clipping is performed on an oversampling rate, however, it produces out-of-band radiation, which requires additional bandpass filter and may also cause peak regrowth. Moreover, it requires larger size \((JN\)-point\) fast Fourier transform (FFT)/IFFT blocks, which increases not only computational complexity, but also hardware costs. To overcome these problems of clipping, H. Ochiai et al. presented that the Nyquist rate clipping combined with the SLM achieves significant PAPR reduction with moderate increment of complexity [15]. Thus, in this paper, we adopt Nyquist rate clipping for the PAPR reduction of OFDM signals at the transmitter. The CR is defined as

\[ CR = 20 \log_{10} \frac{A}{\sigma} \text{dB} \quad (3) \]

where \( \sigma^2 = P_{in} \) is the input power of \( x_m[k] \). Due to the central limit theorem with a relatively large number of subcarriers, say more than 100, the amplitude of \( x_m[k] \) can be assumed to have a Rayleigh distribution with the probability density function

\[ f(|x_m[k]|) = \frac{2|x_m[k]|}{P_m} e^{-|x_m[k]|^2/P_m} \quad (4) \]

Then, the output power of clipped signal is given by

\[ P_{out} = \int_{0}^{\infty} |\bar{x}_m[k]|^2 f(|x_m[k]|) d|x_m[k]| = (1 - \alpha^{-2}) P_{in} \quad (5) \]

where \( \gamma = A/\sigma \). At the receiver, frequency domain channel observation can be expressed as

\[ R_m[n] = H_m[n]X_m[n] + Z_m[n], \quad 0 \leq n \leq N - 1 \quad (6) \]

where \( H_m[n] \) is the channel frequency response (CFR), and \( Z_m[n] \) is the AWGN with variance \( N_0 \).

III. ITERATIVE AMPLITUDE RECONSTRUCTION (IAR) WITH OPTIMUM EQUALIZATION

In this section, we present the IAR for the clipped signal reconstruction, and theoretically compare the IAR with the DAR.

A. The Procedure of IAR

The procedure of IAR is explained as follows with reference to the Fig. 1. Here, the receiver is assumed to know the clipping amplitude \( A \), defined in (2).

1) Frequency domain channel observation, \( R_m[n] \), is obtained by performing FFT on the discrete received samples, \( \{r_m[k]\}_{k=0}^{JN-1} \).

2) Then, the estimate of the clipped sample, \( \hat{x}_m[k] \), is obtained and stored in memory by performing IFFT on \( \{X_m[n]\}_{n=0}^{N-1} \), where

\[ \hat{X}_m[n] = \frac{H_m^*[n]}{|H_m[n]|^2 + N_0/P_{out}} \cdot R_m[n] \]

\[ = W_m[n]R_m[n], \quad 0 \leq n \leq N - 1 \quad (7) \]

\( W_m[n] \) is an MMSE equalizer tap coefficient for the \( n \)th subcarrier, and \((\cdot)^* \) denotes the complex-conjugate operation.

3) Estimate the transmitted symbols \( \{\hat{X}_m^{(I)}[n]\}_{n=0}^{N-1} \) from the \( \{C_m[n], R_m[n]\}_{n=0}^{N-1} \), where \( I \) represents the iteration number and starts with an initial value of \( I = 0 \). \( C_m[n] \) is an optimum equalizer tap coefficient to get the non-clipped OFDM samples from the received signals, which will be detailed at the end of this subsection.

4) IFFT is performed on the decisions in Step 3 to obtain the estimates of the samples before the clipping, thus yielding \( \bar{x}_m^{(I)}[k] \).

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1The guard carriers of OFDM systems provide an inherent oversampling. For simplicity, however, it is not considered in this paper.
5) Clipped signals are detected by comparing the amplitude of \( \hat{x}_m^{(I)}[k] \) to \( A \). Then, the amplitude of clipped signals is reconstructed, and a new sequence \( \{y_m^{(I)}[k]\}_{k=0}^{N-1} \) is generated as

\[
y_m^{(I)}[k] = \begin{cases} \hat{x}_m[k], & |\hat{x}_m[k]| \leq A, \\ |\hat{x}_m[k]| \exp\{\arg(\hat{x}_m[k])\}, & |\hat{x}_m[k]| > A, \\ 0, & 0 \leq k \leq N - 1. \end{cases}
\]

6) The sequence \( \{y_m^{(I)}[k]\}_{k=0}^{N-1} \) is converted to the frequency domain, yielding \( Y_m^{(I)}[n] \), and the transmitted signals \( \{\hat{X}_m^{(I+1)}[n]\}_{n=0}^{N-1} \) are estimated.

7) This completes the \( I \)th iteration, and for more iterations, go back to Step 4 with \( I = I + 1 \).

Since the clipping affects not the phase but the amplitude of OFDM signals, IAR replaces only the amplitude of detected samples in Step 5. This is why we call this algorithm as iterative amplitude reconstruction. From Fig. 1 and the discussion above, each iteration of the amplitude reconstruction requires a single pair of \( N \)-point IFFT/FFT operations.

In IAR and DAR, clipped signals are recovered by using the estimates of clipped and nonclipped OFDM samples, which are \( \tilde{x}_m[k] \) and \( x_m^{(I)}[k] \), respectively. While the DAR uses only one equalizer for recovering these two estimates, the proposed IAR uses two different equalizers, i.e., \( W_m[n] \) and \( C_m[n] \), with an increase of complexity due to the additional equalization. From the (15), the clipped OFDM sample \( \tilde{x}_m[k] \) in (2) can be expressed as the aggregate of the attenuated OFDM signal and the clipping noise \( d_m[k] \) as

\[
\tilde{x}_m[k] = \alpha x_m[k] + d_m[k], \quad 0 \leq k \leq N - 1
\]

where the attenuation factor \( \alpha \) is a function of \( \gamma \) and is given by

\[
\alpha = \frac{E\{x_m[k]\tilde{x}_m[k]\}}{E\{x_m[k]x_m[k]\}} = 1 - e^{-\gamma^2} + \frac{\sqrt{2\pi} \gamma}{2} \text{erfc}(\gamma).
\]

Since \( x_m[k] \) is assumed to be a complex i.i.d. Gaussian random variable with a zero mean, the memoryless property of the system may guarantee \( d_m[k] \) to be a complex i.i.d. random variable with a zero mean, but not Gaussian. The clipping noise \( D_m[n] \) that falls on the \( n \)th subcarrier can be written as

\[
D_m[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_m[k] W_N^{nk}, \quad 0 \leq n \leq N - 1
\]

where \( W_N^{nk} \triangleq e^{-j(2\pi k/N)} \). As the number of subcarriers increases and the CR value decreases, \( \{D_m[n]\}_{n=0}^{N-1} \) approach complex Gaussian random variables with a zero mean. Then, the frequency domain channel observation in (6) can be expressed as

\[
R_m[n] = H_m[n]X_m[n] + Z_m[n]
\]

\[
= H_m[n](\alpha X_m[n] + D_m[n]) + Z_m[n]
\]

\[
= \alpha H_m[n]X_m[n] + Q_m[n],
\]

\[
0 \leq n \leq N - 1
\]

where \( Q_m[n] \) is the sum of \( H_m[n]D_m[n] \) and \( Z_m[n] \). For simplicity, we assume that the input power \( P_{in} \) is equal to 1 without loss of generality. Since \( X_m[n] \), \( D_m[n] \), and \( Z_m[n] \) are uncorrelated and \( E[|D_m[n]|^2] = P_{out} - \alpha^2 P_{in} \), the optimum equalizer tap coefficients, \( \{C_m[n]\}_{n=0}^{N-1} \), are given by

\[
C_m[n] = \frac{(\alpha H_m[n])^*}{|\alpha H_m[n]|^2 + E[|Q_m[n]|^2]}
\]

\[
= \frac{\alpha^2 |H_m[n]|^2 + E[|H_m[n]D_m[n]|^2] + E[|Z_m[n]|^2]}{\alpha H_m[n]}
\]

\[
= \frac{1}{(1 - e^{-\gamma^2}) |H_m[n]|^2 + N_0},
\]

\[
0 \leq n \leq N - 1
\]

based on the MMSE criterion.

### B. Performance Analysis of Clipped OFDM before the first iteration

Since decision error propagates to the next iteration, the reliability of initial estimates considerably affects the overall system performance. To get reliable initial estimates of the nonclipped OFDM samples, we have derived an optimum MMSE equalizer, \( C_m[n] \), in Section III-A. In this subsection, we evaluate the SER performance of clipped OFDM before the first iteration by calculating the means and variances of the equalizer outputs, and show that the proposed equalizer \( C_m[n] \) increases the reliability of the initial estimates compared to the equalizer \( W_m[n] \). For simplicity, 16-ary quadrature amplitude modulation (16-QAM) without channel coding is considered, and the perfect synchronization and carrier recovery at the receiver are assumed. The output of equalizer \( C_m[n] \) in the Step 3 can be written as

\[
C_m[n]R_m[n]
\]

\[
= \frac{\alpha H_m[n](\alpha H_m[n]X_m[n] + H_m[n]D_m[n] + Z_m[n])}{(1 - e^{-\gamma^2}) |H_m[n]|^2 + N_0}.
\]
The means and variances of the equalizer outputs are given by

\[ \mu_c[X_m[n]=S_i] = E[C_m[n]R_m[n]|X_m[n]=S_i] \]
\[ \sigma_c^2[X_m[n]=S_i] = E[|C_m[n]R_m[n]|^2|X_m[n]=S_i] - \mu_c^2[X_m[n]=S_i] \]
\[ = \alpha^2|H_m[n]|^2 \left[ \{1-(e^{-\gamma^2})|H_m[n]|^2 + N_0 \}^2 \right] = \sigma_c^2[n] \]  

(15)

where \( S_i \) are the 16-QAM symbols, and the subscript “c” represents the equalizer \( C_m[n] \). From (15), it is observed that the variance \( \sigma_c^2[n] \) is the same for all \( S_i \).

In order to obtain the SER performance of initial estimates, we derive the SER for the horizontal and vertical 4-ary pulse amplitude modulation (4-PAM) first, and then extend the result into the 16-QAM. Since we have assumed the unit input power, i.e., \( E[|X_m[n]|^2] = 1 \), amplitude levels and decision thresholds of the horizontal and vertical 4-PAMs for 16-QAM are given by \( \left\{ \frac{-3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}} \right\} \) and \( \left\{ \frac{2}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{-1}{\sqrt{10}}, \frac{-2}{\sqrt{10}} \right\} \), respectively. Assuming equally probable symbol alphabets, and from (15), the SER for 4-PAM can be obtained as follows

\[ SER_{4-PAM}[n] = \frac{1}{2} \left\{ Q \left( \frac{3}{\sqrt{10}} \mu_c[n] - \frac{2}{\sqrt{10}} \right) \right\} + \left\{ Q \left( \frac{\frac{1}{\sqrt{10}} \mu_c[n]}{\sigma_c[n]/\sqrt{2}} \right) + Q \left( \frac{-\frac{1}{\sqrt{10}} \mu_c[n]}{\sigma_c[n]/\sqrt{2}} \right) \right\} \]  

(16)

where

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt, \]
\[ \mu_c[n] = \frac{\alpha^2|H_m[n]|^2}{(1-e^{-\gamma^2})|H_m[n]|^2 + N_0}. \]  

(17)

From (16), before the first iteration, the SER performance of clipped OFDM with the equalizer \( C_m[n] \) is given by

\[ SER_{16-QAM} = \frac{1}{N} \sum_{n=0}^{N-1} \left\{ 1 - (1 - SER_{4-PAM}[n])^2 \right\}. \]  

(18)

Without the \( C_m[n] \), the SER performance of clipped OFDM with only the equalizer \( W_m[n] \) can be similarly obtained.

Fig. 2 shows the simulated and analyzed SER performances of the clipped OFDM with equalizers \( C_m[n] \) and \( W_m[n] \) before the first iteration. 1024 subcarriers (N=1024), 5MHz bandwidth, and a six-tap bad urban (BU) channel are considered. Perfect channel state information (CSI) is assumed. As seen in the figure, analyzed and simulated results are in a good agreement, and the optimum equalizer \( C_m[n] \) increases the reliability of initial estimates.

C. Propagation Error of DAR and IAR to the next iteration

Besides the optimum equalizer \( C_m[n] \), another major difference between the DAR and IAR is in the Step 5 of the IAR procedure in Section III-A. In this subsection, we discuss the effectiveness of the Step 5. In the case of DAR, any detection of the clipped sample is replaced with the estimated version, \( \hat{x}_m^{(I)}[k] \). Let the estimate of the time domain sample \( \hat{x}_m^{(I)}[k] \) be the sum of the transmitted sample \( x_m[k] \) and the residual error \( p_m^{(I)}[k] \) as

\[ \hat{x}_m^{(I)}[k] = x_m[k] + p_m^{(I)}[k], \quad 0 \leq k \leq N-1. \]  

(19)

Here, all the power of \( p_m^{(I)}[k] \) propagates to the next iteration in the case of DAR. If we assume perfect channel gains and high SNR, the reconstructed samples of the IAR in Step 5 can be rewritten as

\[ y_m^{(I)}[k] = |\hat{x}_m^{(I)}[k]| \exp\{\arg(\hat{x}_m[k])\} \approx |\hat{x}_m^{(I)}[k]| \exp\{\arg(x_m[k])\} = |\hat{x}_m^{(I)}[k]| \exp\{\arg(x_m[k])\} = x_m[k] + v_m^{(I)}[k], \quad \text{if } |\hat{x}_m^{(I)}| > A \]  

(20)

where \( v_m^{(I)}[k] \) is the error component propagating to the next iteration. Since \( |p_m^{(I)}[k]| \ll |x_m[k]| \) in general, \( v_m^{(I)}[k] \) can be considered as the projection of \( p_m^{(I)}[k] \) onto \( x_m[k] \) (see Fig. 3). For the OFDM systems, frequency selective channel can be modelled as \( N \) parallel independent flat channels. Therefore, the decision error in the frequency domain, \( P_m^{(I)}[n] = X_m^{(I)}[n] - X_m[n] \), can be assumed to be a zero-mean independent random variable such that the time domain error \( p_m^{(I)}[k] \) approaches a circular symmetric zero-mean Gaussian process for large \( N \). As a result, the IAR is more robust against the decision error than the DAR because the power of \( v_m^{(I)}[k] \) is almost half of the \( p_m^{(I)}[k] \).
IV. IAR for STBC/SFBC MIMO-OFDMs

In this section, we extend the IAR for SISO-OFDM into the two-by-two MIMO-OFDMs with spatial diversity, such as STBC and SFBC.

A. IAR for Clipped STBC-OFDM

Let \( \{X_{p,m}[n]\}_{n=0}^{N-1}, \ p = 1, 2, m = 2i, 2i + 1 \), denote the transmitted symbol sequence of the \( p \)th transmit antenna at the \( m \)th OFDM block duration. The STBC encoder generates the coded symbol \( X_{p,m}[n] \) of the \( n \)th subcarrier as follows [3]

\[
\begin{bmatrix}
X_{1,2i}[n] & X_{1,2i+1}[n] \\
X_{2,2i}[n] & X_{2,2i+1}[n]
\end{bmatrix}
\triangleq
\begin{bmatrix}
S_{2i}[n] & S_{2i+1}[n] \\
S_{2i+1}[n] & -S_{2i}[n]
\end{bmatrix},
\]

where \( S_m = [S_m[0], S_m[1], ..., S_m[N-1]] \) is a channel coded symbol vector for the \( m \)th OFDM block duration. The discrete time OFDM samples, \( \{x_{p,m}[k]\}_{k=0}^{N-1} \), are obtained by taking \( N \)-point IFFT on the coded symbols \( \{X_{p,m}[n]\}_{n=0}^{N-1} \). Then, clipping is performed on the IFFT output sequences as in (2), yielding \( \{\tilde{x}_{p,m}[k]\}_{k=0}^{N-1} \). At the receiver, frequency domain channel observation at the \( q \)th receive antenna can be expressed as

\[
R_{q,m}[n] = \frac{2}{\alpha} H_{pq,m}[n] \tilde{x}_{p,m}[n] + Z_{q,m}[n],
\]

where \( H_{pq,m}[n] \) is the CFR between \( p \)th transmit and \( q \)th receive antennas, and \( Z_{q,m}[n] \) is the AWGN with variance \( N_0 \). If we assume that the CFs between two consecutive OFDM symbols are approximately constant, i.e., \( H_{pq,2i}[n] \approx H_{pq,2i+1}[n] \), the STBC combined signals are obtained as [3]

\[
\bar{S}_{2i}[n] = \begin{bmatrix}
\tilde{X}_{1,2i}[n] \\
\tilde{X}_{2,2i}[n]
\end{bmatrix}
\]

\[
\bar{S}_{2i+1}[n] = \begin{bmatrix}
\sum_{q=1}^{2} (H_{1q,2i}[n] R_{q,2i}[n] - \sum_{q=1}^{2} (H_{1q,2i}[n] R_{q,2i+1}[n]) \\
\sum_{q=1}^{2} (H_{2q,2i}[n] R_{q,2i}[n] + H_{1q,2i}[n] R_{q,2i+1}[n])
\end{bmatrix}, \quad 0 \leq n \leq N-1
\]

where \( \Lambda_{2i}[n] = \Lambda_{2i+1}[n] = \sum_{p=1}^{2} H_{pq,2i}[n] Z_{q,2i}[n] = \sum_{q=1}^{2} (H_{1q,2i}[n] Z_{q,2i}[n] - H_{2q,2i}[n] Z_{q,2i+1}[n]), \quad \text{and} \quad Z_{2i+1}[n] = \sum_{q=1}^{2} (H_{1q,2i}[n] Z_{q,2i}[n] + H_{2q,2i}[n] Z_{q,2i+1}[n]). \)

Here, the STBC combined signal \( \tilde{X}_{p,2i}[n] \) can be seen as a frequency domain channel observation of the SISO-OFDM with equivalent CFR of \( \Lambda_{2i}[n] \), since the clipped signals of each transmit antenna, \( \tilde{X}_{1,2i}[n] \) and \( \tilde{X}_{2,2i}[n] \), are completely separated after the STBC combining. Therefore, the clipped STBC-OFDM signals can be easily recovered by using the IAR for SISO-OFDM.

B. IAR for Clipped SFBC-OFDM

The SFBC encoder generates the coded symbol \( X_{p,m}[n] \) for the neighboring subcarriers, \( n = 2v, 2v + 1, v = 0, 1, ..., N/2 - 1 \), as follows [6]

\[
\begin{bmatrix}
X_{1,1}[2v] & X_{1,1}[2v+1] \\
X_{2,1}[2v] & X_{2,1}[2v+1]
\end{bmatrix} \triangleq \begin{bmatrix}
S_{m}[2v] & S_{m}[2v+1] \\
S_{m}[2v+1] & -S_{m}[2v]
\end{bmatrix}.
\]

Similar to the STBC-OFDM, \( \{x_{p,m}[k]\}_{k=0}^{N-1} \) are obtained by taking \( N \)-point IFFT on the symbols \( \{X_{p,m}[n]\}_{n=0}^{N-1} \), and the clipping is performed as in (2). Assuming that the CFs between adjacent subcarriers are approximately constant, i.e., \( H_{pq,m}[2v] \approx H_{pq,m}[2v+1] \), the SFBC combined signals at the receiver are obtained as

\[
\begin{bmatrix}
\bar{S}_{m}[2v] \\
\bar{S}_{m}[2v+1]
\end{bmatrix} = \begin{bmatrix}
\sum_{q=1}^{2} (H_{1q,m}[2v] R_{q,m}[2v] - H_{2q,m}[2v] R_{q,m}^{*}[2v] + 1) \\
\sum_{q=1}^{2} (H_{1q,m}[2v] R_{q,m}^{*}[2v] + H_{2q,m}[2v] R_{q,m}[2v] + 1)
\end{bmatrix}, \quad 0 \leq v \leq N/2 - 1.
\]

Substituting (22), (24) and the frequency domain representation of (9) into (25), we have

\[
\bar{S}_{m}[2v] = \alpha \sum_{p=1}^{2} |H_{pq,m}[2v]|^2 S_{m}[2v] + \sum_{q=1}^{2} |H_{1q,m}[2v]|^2 D_{1,m}[2v] - |H_{2q,m}[2v]|^2 D_{2,m}^{*}[2v] + 1
\]

\[
+ H_{1q,m}[2v] H_{2q,m}[2v] (D_{1,m}[2v] - D_{1,m}^{*}[2v] + 1)
\]

\[
+ H_{1q,m}[2v] Z_{q,m}[2v] - H_{2q,m}[2v] Z_{q,m}^{*}[2v] + 1.
\]

In order to recover the clipped signals of each transmit antenna, \( \tilde{X}_{1,1}[n] \) and \( \tilde{X}_{2,1}[n] \) are required at the receiver like...
the STBC-OFDM. However, in the case of SFBC-OFDM, it is difficult to derive those signals from the \{\tilde{S}_m[2v], \tilde{S}_m[2v+1]\}, since \(D_{1,m}[2v] \neq -D_{2,m}[2v + 1]\) and \(D_{1,m}[2v + 1] \neq D_{2,m}[2v]\), which means that the clipping destroys the orthogonality of SFBC-OFDM transmitted signals. In other words, \(X_{1,m}[n]\) and \(\tilde{X}_{2,m}[n]\) can not be completely separated at the receiver, because \(\tilde{S}_m[2v]\) and \(\tilde{S}_m[2v+1]\) in (26) are mixed with the clipping noises from each transmit antenna. Therefore, it is difficult to obtain robust estimates of the clipped and nonclipped OFDM samples for recovering the amplitude of clipped signals.

V. PROPOSED SFBC-OFDM WITH AMPLITUDE CLIPPING

In Section IV-B, we have shown that the IAR for SISO-OFDM cannot be directly applied to the SFBC-OFDM, because the clipping destroys the orthogonality of SFBC-OFDM transmitted signals. In this section, we propose a new SFBC transmitter for clipped OFDM and its signal reconstruction method at the receiver.

A. Proposed Clipping for SFBC-OFDM

By separating \(\{X_{1,m}[n]\}_{n=0}^{N-1}\) of the SFBC-OFDM into even and odd elements, time domain signals of the first antenna can be written as

\[
x_{1,m}[k] = \frac{1}{N} \sum_{n=0}^{N-1} X_{1,m}[n] W^{-nk} = \frac{1}{N} \sum_{v=0}^{(N/2)-1} \left( S_m[2v] + W^{-k}s_m[2v+1] \right) W^{-vk}
\]

(27)

where \(s_m^e[k]\) and \(s_m^o[k]\) are represented as

\[
s_m^e[k] = \sqrt{\frac{2}{N}} \sum_{v=0}^{(N/2)-1} S_m[2v] W^{-vk},
\]

\[
s_m^o[k] = \sqrt{\frac{2}{N}} \sum_{v=0}^{(N/2)-1} S_m[2v+1] W^{-vk},
\]

Since \(s_m^e[k]\) and \(s_m^o[k]\) are periodic in \(k\) with period \(N/2\), we can replace them with \(s_m^e[(k)(N/2)]\) and \(s_m^o[(k)(N/2)]\). Here, we clip \(s_m^e[k](N/2)\) and \(s_m^o[k](N/2)\) instead of \(x_{1,m}[k]\). Then, the transmitted signals of the first antenna can be written as

\[
\tilde{x}_{1,m}[k] = \frac{1}{\sqrt{2}} \left( \tilde{s}_m^e[(k)(N/2)] + W^{-k}\tilde{s}_m^o[(k)(N/2)] \right),
\]

(29)

Because conjugate operation does not change the PAPR properties, transmitted signals of the second antenna can be derived from the transmitted antenna signals using the SFBC in (24) and discrete Fourier transform (DFT) symmetry property [23], i.e., \(s^*[k]\) or \(s[k]\), \(n = 0, 1, ..., N - 1\), as follows

\[
\tilde{x}_{2,m}[k] = \frac{1}{\sqrt{2}} \left( \tilde{s}_m^e[-(k)(N/2)] - W^{-k}\tilde{s}_m^o[-(k)(N/2)] \right),
\]

(30)

This implies that only \(N/2\) multiplications and \(N\) additions are required to derive \(\tilde{x}_{2,m}[k]\) from \(\tilde{x}_{1,m}[k]\). The block diagram of proposed SFBC transmitter for clipped OFDM is shown in Fig. 4 (a), which has approximately half the computational complexity of the conventional SFBC-OFDM transmitter, especially when the number of subcarriers is large. Note that the proposed clipping preserves the orthogonality of transmitted signals, and the clipped signals, \(\tilde{s}_m^e[k]\) and \(\tilde{s}_m^o[k]\), can be completely separated after the SFBC combining at the receiver, while the addition of separately clipped signals increases the PAPR of transmitted signals. Fig. 5 shows the PAPR complementary cumulative distribution functions (CCDFs) of clipped OFDM and proposed clipped SFBC-OFDM with \(N=128\) at \(CR=0\) dB. The ideal bandlimited analog OFDM signals are approximated by oversampling the discrete signals by a factor of sixteen. It is observed that the PAPR of proposed SFBC-OFDM is approximately 1 dB higher than the clipped OFDM at CCDF \(= 10^{-3}\).

B. IAR for Proposed SFBC-OFDM

In this subsection, we describe the IAR for the proposed SFBC-OFDM. The procedure of the proposed receiver is explained as follows with reference to Fig. 4 (b).
OFDM. Fig. 5. PAPR CCDFs of the clipped OFDM and proposed SFBC-OFDM.

1) From the received signals, SFBC combined signals are obtained as

\[
\left[ \begin{array}{c}
\tilde{S}_m[2v] \\
\tilde{S}_m[2v+1]
\end{array} \right] = \left[ \begin{array}{c}
\sum_{q=1}^{2} \left( H_{1,q,m}[2v] R_{q,m}[2v] - H_{2,q,m}[2v] R_{q,m}^*[2v+1] \right) \\
\sum_{q=1}^{2} \left( H_{2,q,m}[2v] R_{q,m}^*[2v] + H_{1,q,m}[2v] R_{q,m}[2v+1] \right)
\end{array} \right]
\approx \Lambda_m[2v] \tilde{S}_m[2v] + Z_m[2v] \\
\Lambda_m[2v+1] \tilde{S}_m[2v+1] + Z_m'[2v+1],
\]

\[0 \leq v \leq N/2 - 1\] (31)

where \(\Lambda_m[2v] = \sum_{q=1}^{2} \left| H_{q,m}[2v] \right|^2, \quad Z_m[2v] = \sum_{q=1}^{2} \left( H_{q,m}[2v] Z_{q,m}[2v] - H_{2,q,m}[2v] \right), \quad Z_m'[2v+1] = \sum_{q=1}^{2} \left( H_{2,q,m}[2v] Z_{q,m}^*[2v] + H_{1,q,m}[2v] Z_{q,m}[2v+1] \right)\).

2) Estimates of the clipped samples, \(\hat{s}_{m}^{e}[k]\) and \(\hat{s}_{m}^{o}[k]\), are obtained and stored in memory by performing two \(N/2\)-point IFFTs on even and odd elements of the \(\{ \tilde{S}_m[n] \}_{n=0}^{N-1}\), where

\[\hat{s}_{m}[n] = \frac{\sigma_S^2}{\Lambda_m[n] \sigma_S^2 + N_0} \tilde{S}_m[n] = W_m[n] \tilde{S}_m[n], \quad 0 \leq n \leq N - 1.\] (32)

\(\{ W_m[n] \}_{n=0}^{N-1}\) are the MMSE equalizer tap coefficients for the clipped OFDM samples.

3) Estimate the channel coded OFDM symbol vector \(\{ \hat{S}_m^{(l)}[n] \}_{n=0}^{N-1}\) from the \(\{ C_m[n] \tilde{S}_m[n] \}_{n=0}^{N-1}\), where \(C_m[n]\) is the optimum MMSE equalizer tap coefficient for the nonclipped OFDM samples, and the number of iteration starts with \(I = 0\). Since the SFBC combined signals in Step 1 can be represented as

\[\hat{S}_m[n] \approx \Lambda_m[n] \tilde{S}_m[n] + Z_m'[n] = \Lambda_m[n] (\alpha \tilde{S}_m[n] + D_m[n]) + Z_m'[n], \quad 0 \leq n \leq N - 1\] (33)

and \(\tilde{S}_m[n], D_m[n], \) and \(Z_m'[n]\) are uncorrelated, \(\{ C_m[n] \}_{n=0}^{N-1}\) are obtained as follows

\[C_m[n] = \frac{\alpha \Lambda_m[n] \sigma_S^2}{(\alpha \Lambda_m[n] \sigma_S^2 + E[|\Lambda_m[n] D_m[n]|^2] + E[|Z_m'[n]|^2]}
\]

\[= \frac{(\alpha \Lambda_m[n] \sigma_S^2 + \Lambda_m[n] \sum_{q,m} |1 - e^{-j \gamma_q m} \sigma_S^2| + \Lambda_m[n] N_0)}{(1 - e^{-2 \gamma^2}) \Lambda_m[n] \sigma_S^2 + N_0}, \quad 0 \leq n \leq N - 1\] (34)

where \(\sigma_S^2(= E[|S_m[n]|^2])\) is the power of \(S_m[n]\).

4) \(\hat{s}_{m}^{(l)}[k]\) and \(\hat{s}_{m}^{(0)}[k]\) are obtained by performing two \(N/2\)-point IFFTs on even and odd elements of the \(\hat{S}_m^{(l)}[n]\), and the clipped signals are detected by comparing the amplitude of \(\hat{s}_{m}^{(l)}[k]\) and \(\hat{s}_{m}^{(0)}[k]\) to \(A\). Then, the clipped signals are reconstructed, and two new sequences with block length \(N/2\), \(\{ y_m^{(i)}[k] \}_{k=0}^{N/2-1}\) and \(\{ y_m^{(o)}[k] \}_{k=0}^{N/2-1}\), are generated as

\[\{ y_m^{(e)}[k] \} = \begin{cases} \exp(\arg(\hat{s}_{m}^{(e)}[k])), & |\hat{s}_{m}^{(e)}[k]| \leq A, \\
0, & |\hat{s}_{m}^{(e)}[k]| > A, \end{cases}\]

\[0 \leq k \leq N/2 - 1.\] (35)

5) \(y_m^{(e)}[k]\) and \(y_m^{(o)}[k]\) are converted to the frequency domain, yielding \(Y_m^{(e)}[2v]\) and \(Y_m^{(o)}[2v+1]\), and the channel coded OFDM symbol vector \(\{ \hat{S}_m^{(l+1)}[n] \}_{n=0}^{N-1}\) is estimated.

6) This completes the \(I\)th iteration, and for more iterations, go back to Step 4 with \(I=I+1\).

From Fig. 4 (b) and the discussion above, each iteration requires two pairs of \(N/2\)-point IFFT/FFT operations, which has even less computational complexity than that of the IAR for SISO-OFDM. Note that we have derived the \(\{ C_m[n] \}_{n=0}^{N-1}\) in Step 3 to get reliable estimates of the nonclipped OFDM samples before the first iteration, and replace only the amplitude of clipped samples in Step 4, since the clipping affects not the phase but the amplitude of the signals.

VI. CHANNEL ESTIMATION AND SIMULATION RESULTS

A. Block-type Channel Estimation for Clipped OFDM

In this paper, the channel estimation is based on the method proposed in [24], but without the recursive process for reducing the computational complexity. In the SISO-OFDM, we have used the Chu sequence as a pilot symbol block, which has constant amplitude in both frequency and time domains [25]. Due to the unit PAPR property, utilizing the Chu sequence as a pilot symbol block precludes the problem of nonlinear distortion caused by the clipping when the CR value is greater than 0dB. The \(n\)th element of a length-\(N\) Chu sequence is given by

\[C_n[n] = \begin{cases} e^{j \pi r n^2 / N}, & \text{for even } N \\
0, & \text{for odd } N \end{cases}\] (36)

where \(r\) is relatively prime to \(N\).

Channel estimation for the MIMO-OFDM can be done by transmitting a pilot symbol block from one antenna at a time while the remaining transmit antennas are idle. This method, however, becomes inefficient when the number of transmit
antennas is large. To cope with this problem, we design orthogonal pilot symbol blocks for each transmit antenna. Using this design, only one OFDM block is needed to estimate the entire MIMO channels. In the case of two transmit and two receive antennas, a simple way of designing such pilot symbol blocks is by sending pilot symbols on even subcarriers of the first antenna, and zeros on those subcarriers for the second antenna. For the odd subcarriers, the second antenna sends pilot symbols, while the first antenna transmits zeros. In order to have constant amplitude pilot sequences in time domain, like the SISO-OFDM, the Chu sequence with length-\(N/2\) can be used as pilot symbols for the MIMO-OFDM as follows

\[
X_1^{p} = [ C_{N/2}[0], 0, C_{N/2}[1], 0, ..., C_{N/2}[N/2-1], 0],
\]

\[
X_2^{p} = [0, C_{N/2}[0], 0, C_{N/2}[1], 0, ..., C_{N/2}[N/2-1]]
\]

(37)

where the subscript “\(p\)” represents the pilot symbol block. Note that \(x_{1,m}^{p}\) (time domain representation of \(X_1^{p}\)) is a simple repetition of the time domain Chu sequence with length-\(N/2\), and \(x_{2,m}^{p}\) is the phase rotated version of the \(x_{1,m}^{p}\). Therefore, both \(x_{1,m}^{p}\) and \(x_{2,m}^{p}\) have the desirable unit PAPR property. The nulled subcarriers are reconstructed at the receiver by using the interpolation algorithm described in [24].

### B. Simulation Results

The BER performance of IAR for clipped SISO and MIMO-OFDMs are investigated through computer simulations. We consider coded OFDMs with 1024 subcarriers (\(N=1024\)), 16-QAM constellation, 5MHz bandwidth, and a 1/2-rate convolutional code with constraint length of 3. Every frame consists of one pilot symbol block and 10 OFDM data blocks. Decoding is carried out by using the maximum a posteriori (MAP) algorithm [26], which minimizes the bit error probability. Cyclic prefix is appended to each OFDM block to eliminate the inter-symbol interference caused by multipath channels, and 6-tap typical urban (TU) and BU channels with 0.0001 normalized Doppler frequency \(f_d\) are used. The total transmit power from MIMO-OFDM antennas is set to the same as the power of SISO-OFDM, and MMSE equalizer is applied to the nonclipped OFDM systems.

![Fig. 6. BER performance of the IAR for SISO-OFDM over BU channel (\(N=1024\), 16-QAM, CR=0dB, \(f_d\)=0.0001).](image)

Fig. 6 shows the performance of IAR for SISO-OFDM with and without \(C_m[n]\) over the BU channel (see Step 3 of the IAR in Section III-A). Only \(W_m[n]\) is used for the system without optimum equalization of \(C_m[n]\). It is observed that the optimum equalization method increases the reliability of the initial estimates, and the receiver with the optimum equalizer dramatically improves the overall system performance. With only two iterations, the BER of IAR with \(C_m[n]\) approaches the nonclipped system at SNR=18dB. In order to explain the effect optimum equalization more clearly, the mean squared errors (MSEs) of the clipped system with and without \(C_m[n]\) are given in Fig. 7. The MSEs are measured at the outputs of the equalizers. It shows that the \(C_m[n]\) reduces the MSEs about 2dB at all subcarriers except the nulled subcarriers. The reduction of the MSE leads to a reliable estimation of the transmitted symbols before the first iteration, which facilitates the iterative IAR process. In Fig. 8, the performance of IAR and DAR over TU channel is compared, where the optimum equalization is used for both of the schemes. The IAR outperforms the DAR. As analyzed in Section III-C, the
difference of performance between the IAR and DAR is due to the residual error propagating to the next iteration (see Step 5 of the IAR in Section III-A). In Fig. 9, the results of Figs. 6, 7 and 8 are summarized by the BER curves for different CRs. The advantage of \( C_m[n] \) is maintained through all CR values, and highlighted especially at low CR values. Due to the propagation error to the next iteration, the DAR works well only at high CR values, while the IAR approaches the nonclipped SISO-OFDM through all CR values. In Figs. 6 and 8, the performance of SISO-OFDMs with the channel estimation is also given. It is observed that there is almost no performance degradation compared to the systems with perfect CSI.

Fig. 10 shows the BER performance of IAR for STBC-OFDM. The performance of clipped 2\( \times 2 \) STBC-OFDM without the signal reconstruction is even worse than that of the 2\( \times 1 \) STBC-OFDM. However, with three iterations, the proposed receiver approaches the nonclipped 2\( \times 2 \) STBC-OFDM with only 0.3dB SNR gap at BER=\( 10^{-6} \). The performance of IAR for SFBC-OFDM is shown in Fig. 11. It is observed that the optimum equalizer \( C_m[n] \) increases the reliability of initial estimates, and thus the IAR for proposed SFBC-OFDM significantly improves the overall system performance. The result of IAR for conventional SFBC-OFDM in Section IV-B is also given in the figure. For the amplitude reconstruction, \( \tilde{X}_{1,m}[v] \) and \( \tilde{X}_{2,m}[v] \) are obtained from the SFBC encoding of \( S_m[2v] \) and \( S_m[2v+1] \) in (28). Due to the destruction of orthogonality, it shows approximately one order of BER degradation compared to the proposed SFBC-OFDM at SNR=12dB. In addition, it requires two pairs of \( N \)-point IFFT/FFT operations per each iteration, while the proposed receiver requires two pairs of \( N/2 \)-point IFFT/FFT operations. In Figs. 10 and 11, the performance of MIMO-OFDMs with channel estimation is also given. Note that the clipped systems with channel estimation have slight performance degradation compared to the systems with perfect CSI, and highly accurate channel estimation for the MIMO-OFDM employing a clipping can be done by using the Chu sequence design described in Section IV-A, which has the desirable unit PAPR property.

VII. CONCLUSIONS

We first proposed clipped signal reconstruction methods for MIMO-OFDMs based on the IAR. Since the IAR is an iterative technique, the performance of IAR largely depends on the reliability of initial estimates and the propagation error to the next iteration. Theoretical analysis showed that the optimum equalization of IAR increases the reliability of initial estimates and its amplitude reconstruction halves the power of propagation error to the next iteration compared to the DAR. Further, we showed that the IAR can not be directly employed for the SFBC-OFDM due to the dependency of sequences over each transmit antenna, while it can be easily applied to the STBC-OFDM. We proposed a new SFBC transmitter for clipped OFDM, which has approximately half the computational complexity of conventional SFBC-OFDM and can also be applied to nonclipped OFDM. The proposed clipping preserves the orthogonality of transmitted signals, and the clipped signals are iteratively recovered at the receiver. Finally, we presented that accurate channel estimation of the

![Fig. 9. BER performance comparison of the IAR and DAR over BU channel (N=1024, 16-QAM, SNR=16dB, \( f_d t_s=0.0001 \)).](image)

![Fig. 10. BER performance of the IAR for STBC MIMO-OFDM over BU channel (N=1024, 16-QAM, CR=0dB, \( f_d t_s=0.0001 \)).](image)

![Fig. 11. BER performance of the IAR for SFBC MIMO-OFDM over BU channel (N=1024, 16-QAM, CR=0dB, \( f_d t_s=0.0001 \)).](image)
clipped OFDM systems can be done with a sequence, which has constant amplitude in both frequency and time domains. Although we focused on two transmit antennas, the proposed techniques can be easily extended to quasi-orthogonal signal designs with four and eight transmit antennas [27]. Extensive simulation results show that the proposed receivers effectively recover the amplitude of clipped OFDMs with a moderate computational complexity.

REFERENCES


