Iterative Multiuser Detection with Spectral Efficient Relaying Protocols for Single-Carrier Transmission

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Abstract—In this paper, we present an iterative multiuser detection scheme using frequency-domain equalization (IMD-FDE), which can be incorporated into a single carrier frequency division multiple access (SC-FDMA) framework. The IMD-FDE scheme employs a different interleaver for each user at the transmitter, and removes multiuser interference with an iterative detection method in the frequency domain at the receiver. The extrinsic information transfer (EXIT) charts and bit-error rate (BER) plots are presented to investigate the iterative behavior of the IMD-FDE over additive white Gaussian noise and multipath fading channels. In a conventional time-division multiple-access (TDMA)-based relay protocol, cooperative transmission over two time slots leads to a 50% loss in spectral efficiency. We propose two spectral efficient relay protocols, where multiple sources access a relay, destination, or both simultaneously. We also present a new iterative detection scheme that effectively removes multiuser interferences for the proposed relay protocols. With the proposed protocols and the aid of iterative multiuser detection scheme, the proposed relay-assisted SC-FDMA with IMD-FDE can obtain diversity gain without sacrificing spectral efficiency and approaches the single-user bound over time-varying and frequency-selective fading relay channels.

Index Terms—Single carrier frequency division multiple access (SC-FDMA), multiuser detection, iterative detection, relay.

I. INTRODUCTION

In various fora and organizations for wireless research, such as the Wireless World Research Forum (WWRF) and International Telecommunication Union (ITU), there have been active discussions about 4G or beyond 3G systems to be deployed around 2010 [1]. Single carrier frequency-domain equalization (SC-FDE) has similar performance and essentially the same overall complexity as orthogonal frequency division multiplexing (OFDM) [2], [3]. Recently, the SC-FDE has drawn great attention as an alternative to the OFDM, especially in the uplink communications where lower peak-to-average power ratio significantly benefits the mobile terminal in terms of transmit power efficiency [4]. SC frequency division multiple access (SC-FDMA), which is a combination of FDMA and SC-FDE, is currently a strong candidate for the uplink multiple access scheme in 3rd Generation Partnership Project Long Term Evolution (3GPP-LTE) [5].

As an orthogonal multiple access scheme, SC-FDMA can achieve multiple access interference (MAI)-free transmission by allocating different subcarriers to different users. It is well known that only non-orthogonal multiple access scheme can approach the optimal capacity of multiple access channel [6]-[7]. In particular, the non-orthogonal multiple access scheme with single carrier modulation (i.e., carrierless amplitude and phase modulation (CAP)), which employs interference equalization techniques, was chosen as the physical layer interface standard for wired multiple access applications, such as IEEE 802.9 and the ATM-LAN [7]. For code-division multiple-access (CDMA), each user is assigned a distinct signature sequence (or waveform), which the user employs to modulate and spread the information-bearing signal. The signature sequences allow the receiver to demodulate the message transmitted by multiple users. However, conventional multiuser detection (MUD) techniques for CDMA are highly involved. For example, the well known linear minimum mean square error (MMSE) MUD technique [8] has computational complexity $O(U^2)$ per user, which causes a concern when the number of users $U$ is large. As another non-orthogonal multiple access scheme, interleave-division multiple-access (IDMA) employs distinct interleavers combined with low-rate channel coding and chip-by-chip (CBC) multiuser detection to separate the signals transmitted from different users [9], [10]. However, a rake-type operation of IDMA deteriorates over a multipath channel with large delay spread. To overcome this limitation, iterative multiuser detection with frequency-domain equalization (IMD-FDE) was proposed in [11]. The IMD-FDE achieves better performance than IDMA with CBC detection even for severely frequency-selective channels, while providing considerably lower computational complexity. The IMD-FDE can be incorporated into an SC-FDMA framework, which allows more than one user to share a common set of subcarriers. The SC-FDMA system with IMD-FDE provides the advantages, such as spectral efficiency and flexibility for adjusting the data rate of users, compared with conventional SC-FDMA and IDMA systems, respectively.

The extrinsic information transfer (EXIT) chart proposed in [12] is a convergence prediction method, which uses the mutual information measure to visualize the exchange of extrinsic information during the iterative decoding process. Originally, the EXIT chart analysis was developed to investigate the behavior of concatenated codes. This method was applied to analyze turbo equalization [13] or turbo multiuser detection [14]. In this paper, we analyze the iterative behavior of the IMD-FDE using EXIT charts, over additive white Gaussian noise (AWGN) and multipath fading channels. When
the channel is time-varying, the EXIT chart tunnel becomes narrow or closed. This means that more iterations are needed to approach the optimal performance. Spatial diversity can be used to make the EXIT chart tunnel wide open and to achieve better performance in time-varying and frequency-selective fading channels.

Spatial diversity is a powerful technique to combat the fading effect through the use of multiple antennas at the transmitter and/or receiver side in wireless communications [15]-[17]. In conventional spatial diversity techniques, diversity gain is realized at the physical layer with co-located antennas. Unfortunately, the use of multiple antennas might not be practical at the cellular mobile devices due to the limitation of size and complexity. Cooperative diversity overcomes these problems without the additional complexity of multiple antennas [18]-[25]. Multiple terminals in a network cooperate to form a virtual antenna array, realizing spatial diversity in a distributed fashion, although each of them is equipped with only one antenna. In [21]-[23], multiuser cooperative networks based on full-duplex relaying scheme were proposed for non-orthogonal CDMA systems. In [24] and [25], the authors extended conventional space-time block coded (STBC) and space-frequency block coded (SFBC) SC-FDE systems in a distributed fashion for relay networks, respectively. The relay usually works in half-duplex mode, since the large difference to approach the optimal performance. Spatial diversity can realize cooperative diversity gain. The EXIT Chart analyses and simulation results are given and discussed in Section IV. Section V gives the conclusions.

II. ITERATIVE MULTIUSER DETECTION FOR SC-FDMA

In the SC-FDMA framework, the discrete Fourier transform (DFT) and inverse DFT (IDFT) blocks are employed at the transmitter. These blocks are used to allocate transmit symbols to a subcarrier set of each user. There are two subcarrier mapping modes: distributed subcarrier mapping and localized subcarrier mapping. In the distributed subcarrier mapping mode, the DFT outputs of input data are allocated over the entire bandwidth with zeros occupying the unused subcarriers, whereas consecutive subcarriers are occupied by the DFT outputs of input data in the localized subcarrier mapping mode. The IMD-FDE scheme can be applied to two different subcarrier mapping modes in a similar manner. For simplicity, we do not take the DFT and IDFT blocks into account in the following iterative procedures [11]:

- To perform MMSE equalization, the mean \(x_n\) and variance \(\sigma_n^2\) of transmitted symbols are required. These mean and variance are functions of a priori log likelihood ratios (LLRs) obtained from MAP decoder. For the initial iteration, the a priori information provided by the decoder is not available, and thus the mean and variance of transmitted symbols are set to zero and one, respectively.

- Using the mean values, multiuser interference cancellation (MIC) for the \(u\)th user is performed as \(Z_u = R - \sum_{u' \neq u} \Lambda_{u,u'} F_x u_{u'}\). The frequency-domain estimates are obtained as \(\hat{X}_u = G_u^H Z_u = (G_u^H \Lambda_u - \frac{1}{2} \text{trace}(G_u^H \Lambda_u)) F_x u\). The coefficients of MMSE equalizer are given by

\[
G_u(k) = \frac{\Lambda_u(k)}{\sum_{u' \neq u} \nu_u |\Lambda_{u,u'}(k)|^2 + \sigma_n^2}, \quad k = 0, 1, \ldots, N-1.
\]

The equalized symbols are transformed back to the time-domain by an inverse FFT (IFFT) operation. Using the output of the IFFT operation, we can obtain the extrinsic LLRs \(\{L_{0}^{E}(c_u(m))\}_{m=0}^{2N-1}\) fed to the maximum a posteriori (MAP) decoder. The MAP decoder computes the extrinsic information for both the coded and decoded bits. The extrinsic information \(L_{0}^{E}(c_u(l))\) of the coded bit is used as the a priori information of the multiuser interference canceller and the one-tap MMSE equalizer.

III. SPECTRAL EFFICIENT RELAY-ASSISTED SC-FDMA

A. Protocols and System Models

We assume that the half-duplex constraint (either transmit or receive, but not both) is imposed on the cooperating
nodes. In the time-division multiple-access (TDMA)-based relay protocols [20], [24], [25], cooperative transmission over two time slots leads to a 50% loss in spectral efficiency. We present two new spectral efficient relaying protocols (see Fig. 1). For simplicity, we take two-source case as an example of the proposed scheme. The proposed scheme can be easily extended to the general case of multiple sources, as seen in Section IV. In the proposed protocols, two sources access a relay, destination, or both simultaneously. In Fig. 1(a), during the first (third) phase, two sources communicate with the relay simultaneously, while the destination does not receive the direct signal from the sources. During the second (fourth) phase, the relay and sources communicate with the destination. In Fig. 1(b), during the first phase, two sources communicate with the relay and the destination simultaneously. During the second phase, only the relay communicates with the destination. The protocol in Fig. 1(a) is motivated by the practical consideration that the destination may be engaged in data transmission to another terminal over the network. In a scenario where the source engages in data reception from another terminal in the network over the first (third) phase [20], [24]. Similarly, the protocol in Fig. 1(b) is logical in a scenario where the source engages in data reception from another terminal in the network over the second phase. The protocols in Figs. 1(a) and (b) convert the spatially distributed antenna systems into effective multiple-input-single-output (MISO) and single-input multiple-output (SIMO) channels, respectively. For the sake of convenience, we call the protocols in Figs. 1(a) and (b) as MISO and SIMO protocols throughout this paper, respectively. With the proposed protocols and the aid of iterative multiuser detection, the spectral loss can be recovered. We consider all underlying links experience frequency selective fading. The CIR for the transmitting node (A)-to-receiving node (B) is given by \( h_{AB} = [h_{AB}(0) \ h_{AB}(1) \ \cdots \ h_{AB}(\ell_{AB})]^T \), where \( \ell_{AB} \) denotes the CIR length. In this paper, the subscripts \( S_u, R \) and \( D \) stand for the \( u \)th source, relay, and destination nodes, respectively.

1) Relay-Assisted SC-FDMA with the MISO Protocol: We present a distributed STBC scheme for relay-assisted SC-FDMA, as an example of the proposed MISO protocol. Figure 2(a) depicts a transmitter block diagram of the proposed relay-assisted SC-FDMA system with STBC transmission. The STBC encoder is used only in the third and fourth phases. The coded bits are independently interleaved using a user-specific interleaver, and then they are mapped to QPSK symbol sequences with a size of \( N \). We assume that the CIRs are constant over four consecutive blocks and vary independently every four blocks. In the first time slot, a CP of length \( \ell = \text{max}(\ell_{S_R}, \ell_{S_D}) \) is appended in the head of \( \{x_u(n)\}_{n=0}^{N-1} \), \( u = 1, 2 \), and then the symbol block is transmitted over a fading channel. At the relay, removing the CP, the \( j \)th received signal is given by

\[
r_j^R = \sqrt{E_{S_R} r} H_{S_R}^j x^1 + \sqrt{E_{S_D} r} H_{S_D}^j x^2 + w_j^R.
\]

where \( x^1 = [x^1(0) \ x^1(1) \ \cdots \ x^1(N-1)]^T \), \( u = 1, 2 \), denotes the first transmit vector of the \( u \)th user. \( E_{S_R} \) represents the average energy available at the receiving node \( B \) and includes path loss and shadowing effects in the \( A \rightarrow B \) link for simplicity. \( H_{AB} \) is an \( N \times N \) circulant channel matrix with entries \( [h_{AB}]_{k,l} = h_{AB}((k-l) \text{ mod } N) \). \( w_j^R \) is a complex AWGN vector with covariance matrix of \( \sigma_w^2 \). The received signal, \( r_j^R \), is normalized as

\[
r_j^R \triangleq \sqrt{2/(E_{S_R} + E_{S_D} + \sigma_w^2)} r_j^R \triangleq \gamma_j r_j^R.
\]

to ensure a balance of power at the destination. Then, the \( j \)th transmit signal of the relay can be represented as

\[
x_j^R = \gamma_j x_j^R \sqrt{E_{S_R} r} H_{S_R}^j x^1 + \gamma_j \sqrt{E_{S_D} r} H_{S_D}^j x^2 + \gamma_j w_j^R.
\]

In the second time slot, the sources and relay transmit \( x^2 \) after appending CPs with length \( \ell = \text{max}(\ell_{S_D}, \ell_{S_R}, \ell_{RD}) \), respectively. At the destination, removing the CP, the \( j \)th received signal is given by

\[
r_j^D = \sqrt{E_{S_D} r} H_{S_D}^j x^2 + \sqrt{E_{S_R} r} H_{S_R}^j x^1 + \sqrt{E_{RD} r} H_{RD}^j w_j^R + w_j^D
\]

where \( w_j^D \) is a complex AWGN vector with each entry having a zero-mean and variance of \( \sigma_w^2 \). The \( w_j^D \) is a complex AWGN vector with each entry having a zero-mean and variance of \( \sigma_w^2 \). In the third and fourth time slots, the transmit blocks for \( S \rightarrow R \) and \( S \rightarrow D \) links are generated by the STBC encoder as follows:

\[
x_j^3 = -Jx_u^2, \quad \text{and} \quad x_j^4 = Jx_u^1, \quad u = 1, 2,
\]

where \( J \) is a \( N \times N \) permutation matrix, which performs a reversed cyclic shift, i.e., for a vector \( x \), the \( n \)th element of \( Jx \) is equal to \( x((n-n)\text{ mod } N) \). At the relay and destination, removing the CP, the \( (j+1) \)th received signals \( r_{j+1}^R \) in the third time slot and \( r_{j+1}^D \) in the fourth time slot are given by

\[
r_{j+1}^R = \sqrt{E_{S_R} r} H_{S_R}^{j+1} (-Jx_u^2) + \sqrt{E_{S_D} r} H_{S_D}^{j+1} (-Jx_u^1) + w_{j+1}^R,
\]

\[
r_{j+1}^D = \sqrt{E_{S_D} r} H_{S_D}^{j+1} x^2 + \sqrt{E_{RD} r} H_{RD}^{j+1} w_{j+1}^R + w_{j+1}^D.
\]
where $x_R^{j+1} = \gamma_R x_R^j$. We assume that the CIRs over two consecutive blocks are approximately constant, i.e.,

$$H_{S,R} \triangleq H_{S,R}^2 \approx H_{S,R}^1 \triangleq H_{S,D}^1 \approx H_{S,D}^1, \quad u = 1, 2; \quad H_{RD} \triangleq H_{RD}^1 \approx H_{RD}^2. \quad (9)$$

With the normalization as in [20], $r_D^j$ and $r_D^{j+1}$ can be rewritten as

$$r_D^j = \gamma_S R_D H_{RD} H_{S,R} X_1^j + \gamma_S D H_{S,D} X_1^j + \gamma_S D H_{S,D} X_2^j + w_D^j,$$

$$r_D^{j+1} = \gamma_S D H_{S,D} (J x_1^j) + \gamma_S R_D H_{RD} H_{S,R} (J x_2^j) + \gamma_S D H_{S,D} (J x_2^j) + w_D^{j+1},$$

where

$$\gamma_{S,D} = \eta \sqrt{E_{S,D} (E_{S,R} + ES_R + \sigma^2_w)}, \quad u = 1, 2; \quad \gamma_{S,R_D} = \eta \sqrt{2 E_{S,D} E_{RD}}, \quad u = 1, 2; \quad w_D^m = \eta \sqrt{E_{S,D} + E_{S,R} + \sigma^2_w} (w_D^m + \sqrt{E_{RD} H_{RD} w_R^m}), \quad m = j, j + 1; \quad \eta = \frac{1}{\sqrt{E_{S,R} + E_{S,R} + \sigma^2_w + 2 E_{RD} \sum_{l=0}^{RD} |h_{RD}(l)|^2}}. \quad (10)$$

Then, by performing FFT on $r_D^j$ and $r_D^{j+1}$, $R_D^j = \mathcal{F}[r_D^j]$ and $R_D^{j+1} = \mathcal{F}[r_D^{j+1}]$ can be obtained as follows:

$$R_D^j = \gamma_S R_D \Lambda_S R_D X_1^j + \gamma_S D \Lambda_S D X_1^j + \gamma_S R_D \Lambda_S R_D X_2^j + W_D^j,$$

$$R_D^{j+1} = \gamma_S D \Lambda_S D X_1^{j+1} + \gamma_S R_D \Lambda_S R_D X_2^{j+1} + \gamma_S D \Lambda_S D X_2^{j+1} + W_D^{j+1},$$

where $X_1^p = \mathcal{F} x_u p, p = 1, 2$, and $W_D^m = \mathcal{F} w_D^m, m = j, j + 1$. In (11), $\Lambda_S, D = F H_{S,D} F^{-1}, \Lambda_{RD} = F H_{RD} F^{-1}$, and $\Lambda_{S,R_D} = \Lambda_{S,D} H_{RD} \Lambda_{R,D}, u = 1, 2, 2$.

2) Relay-Assisted SC-FDMA with the SIMO Protocol:

Figure 2(a), except STBC encoder, depicts a transmitter block diagram of the proposed relay-assisted SC-FDMA system with the SIMO protocol. We assume that the CIRs are constant over two consecutive blocks and vary independently every two blocks. In the first time slot, a CP of length $\ell = \max(\ell_{S,R}, \ell_{S,D}, \ell_{S,R}, \ell_{S,D})$ is appended in the head of $\{x_u(n)\}_{n=1}^{N-1}$, $u = 1, 2$, and then the symbol block is transmitted over fading channels. After removing the CP, the received signals at the relay and destination are given by

$$r_D^j = \sqrt{E_{S,D} H_{S,D} x_1 + E_{S,D} H_{S,D} x_2 + w_D},$$

$$r_R = \sqrt{E_{S,R} H_{S,R} x_1 + E_{S,R} H_{S,R} x_2 + w_R}. \quad (12)$$

The received signal, $r_R$, at the relay is normalized as $\bar{r}_R \triangleq \gamma_R r_R$ to ensure balance of power at the destination. Then, the transmit signal of the relay can be represented as

$$x_R = \gamma_R \sqrt{E_{S,R} H_{S,R} x_1 + E_{S,R} H_{S,R} x_2 + \gamma_R w_R}. \quad (13)$$

In the second time slot, the relay transmits $x_R$ after appending CP with length $\ell = \ell_{RD}$. At the destination, removing the CP, the received signal is given by

$$r_D^{j+1} = \sqrt{E_{RD} H_{RD} x_R + w_D^{j+1}} = \gamma_R \sqrt{E_{S,R} E_{RD} H_{RD} H_{S,R} x_1 + \sqrt{E_{RD} H_{RD} w_R^{j+1}} + w_D^{j+1}}, \quad (14)$$

where $w_R^{j+1} = \gamma_R w_R$. With the normalization, $r_D^{j+1}$ can be rewritten as

$$r_D^{j+1} = \gamma_S R_D H_{RD} H_{S,R} x_1 + \gamma_S D H_{S,D} x_2 + w_D^{j+1}, \quad (15)$$

where $\gamma_{S,R,D}, u = 1, 2$, and $w_D^{j+1}$ are defined as in (10).

Then, by performing FFT on $r_D^j$ and $r_D^{j+1}$, $R_D^j (= \mathcal{F}[r_D^j])$ and $R_D^{j+1} (= \mathcal{F}[r_D^{j+1}])$ can be obtained as follows:

$$R_D^j = \sqrt{E_{S,D} \Lambda_S D X_1 + \sqrt{E_{S,D} \Lambda_S D X_2 + \gamma_R w_R^{j+1}}},$$

$R_D^{j+1} = \gamma_S R_D \Lambda_S D X_1 + \gamma_S D \Lambda_S D X_2 + \gamma_R w_R^{j+1}, \quad (16)$$

where $X_1^p = \mathcal{F} x_u p, u = 1, 2, 2$.

B. Iterative Multiuser Detection using FDE for the Proposed Protocols

1) Relay-Assisted SC-FDMA with the MISO Protocol:

Figure 2(b) shows a receiver structure of the proposed relay-assisted SC-FDMA system with the MISO protocol, which exploits the IMD-FDE. After SMIC, the received vectors $Z_1^j$ and $Z_1^{j+1}$ of the first user can be expressed in the frequency domain as

$$Z_1^j = \gamma_S R_D \Lambda_S R_D X_1^j + \gamma_S D \Lambda_S D X_1^j + \gamma_S D \Lambda_S D X_2^j + \mathcal{F} \Lambda_{RD} X_1^j + \gamma_S R_D \Lambda_S R_D X_2^j + \mathcal{F} \Lambda_{RD} X_2^j,$$

$$+ \gamma_S D \Lambda_S D X_2^j,$$

$$+ \gamma_S R_D \Lambda_S R_D X_2^j + \mathcal{F} \Lambda_{RD} X_2^j,$$

$$+ \gamma_S R_D \Lambda_S R_D X_2^j + \mathcal{F} \Lambda_{RD} X_2^j,$$

In (11), $\Lambda_S$ = $\Lambda_{S,D} F^{-1}, \Lambda_{RD} = \mathcal{F} H_{RD} F^{-1}, \Lambda_{S,R_D} = \Lambda_{S,D} H_{RD} \Lambda_{R,D}, u = 1, 2, 2$.

2) Relay-Assisted SC-FDMA with the SIMO Protocol:

The received signal, $r_R$, at the relay is normalized as $\bar{r}_R \triangleq \gamma_R r_R$ to ensure balance of power at the destination. Then, the

transmit signal of the relay can be represented as

$$x_R = \gamma_R \sqrt{E_{S,R} H_{S,R} x_1 + \sqrt{E_{S,R} H_{S,R} x_2 + \gamma_R w_R}}. \quad (13)$$

In the second time slot, the relay transmits $x_R$ after appending CP with length $\ell = \ell_{RD}$. At the destination, removing the CP, the received signal is given by

$$r_D^{j+1} = \sqrt{E_{RD} H_{RD} x_R + w_D^{j+1}} = \gamma_R \sqrt{E_{S,R} E_{RD} H_{RD} H_{S,R} x_1 + \sqrt{E_{RD} H_{RD} w_R^{j+1}} + w_D^{j+1}}, \quad (14)$$

where $w_R^{j+1} = \gamma_R w_R$. With the normalization, $r_D^{j+1}$ can be rewritten as

$$r_D^{j+1} = \gamma_S R_D H_{RD} H_{S,R} x_1 + \gamma_S D H_{S,D} x_2 + w_R^{j+1}, \quad (15)$$

where $\gamma_{S,R,D}, u = 1, 2$, and $w_D^{j+1}$ are defined as in (10).

Then, by performing FFT on $r_D^j$ and $r_D^{j+1}$, $R_D^j (= \mathcal{F}[r_D^j])$ and $R_D^{j+1} (= \mathcal{F}[r_D^{j+1}])$ can be obtained as follows:

$$R_D^j = \sqrt{E_{S,D} \Lambda_S D X_1 + \sqrt{E_{S,D} \Lambda_S D X_2 + \gamma_R w_R^{j+1}}},$$

$$R_D^{j+1} = \gamma_S R_D \Lambda_S D X_1 + \gamma_S D \Lambda_S D X_2 + \gamma_R w_R^{j+1}, \quad (16)$$

where $X_1^p = \mathcal{F} x_u p, u = 1, 2, 2$.

From (18), we can now derive a linear combining receiver under the MMSE criterion. The resultant equation is similar
to that of the maximal ratio combining receiver with the two branch diversity system, which is

$$\mathbf{Z}_1 = [\mathbf{Z}_1^1 \ \mathbf{Z}_1^2]^T = \mathbf{A}_1^H \mathbf{Z}_1$$

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{A}_1^{S,RD} & \mathbf{B}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1^{S,RD} & \mathbf{B}_1 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^1 - \mathbf{X}_1^2 \end{bmatrix} + \begin{bmatrix} \mathbf{A}_1^{S,RD} \mathbf{W}_D^T + \mathbf{A}_1^{S,RD} \mathbf{W}_D^{+1+} \end{bmatrix},$$ (19)

where $\mathbf{A}_1 = |\mathbf{A}_1^{S,RD}|^2 + |\mathbf{A}_1^{S,RD}|^2$ denotes a diagonal matrix whose $(k,k)$ entry $\Lambda_1(k)$ is equal to the weighted sum of the squared $k$th DFT coefficients of the CIRs for $S_1 \rightarrow R$ and $S_1 \rightarrow R \rightarrow D$ links. In (19), $A_2 = \mathbf{A}_1^{S,RD} \mathbf{A}_1^{S,C} + \mathbf{A}_1^{S,RD} \mathbf{A}_1^{S,RD}$, and $B_2 = \mathbf{A}_1^{S,RD} \mathbf{A}_1^{S,RD} - \mathbf{A}_1^{S,QD} \mathbf{A}_1^{S,RD}$. Applying the same method used in Section II to $\mathbf{Z}_1$ and $\mathbf{Z}_1^2$, the coefficients of the one-tap MMSE equalizer for the relay-assisted STBC system are obtained as follows:

$$G_1^1(k) = \frac{\Lambda_1^1}{\nu_k^1 |\Lambda_1^1|^2 + \nu_k^2 |\Lambda_1^2|^2 + \nu_k^2 |\Lambda_1^{+2}|^2 + \sigma_1^2},$$

$$G_1^2(k) = \frac{\Lambda_1^2}{\nu_k^1 |\Lambda_1^1|^2 + \nu_k^2 |\Lambda_1^2|^2 + \nu_k^2 |\Lambda_1^{+2}|^2 + \sigma_1^2},$$

where $\nu_k = \frac{1}{\nu_{k}}$ for all $k, u = 1, 2, p = 1, 2$, and $\mathbf{V}_u = \text{Cov}(\mathbf{x}_u, \mathbf{x}_{u+1}) = \text{Diag}(\nu_{0,u}, \nu_{1,u}, \ldots, \nu_{N-1,u})$ represents the covariance matrix of the transmit vector $\mathbf{x}_u$. We can also employ the procedures used in Section II to obtain the frequency-domain estimates $\{\hat{X}_k^1(n)\}_{k=0}^{N-1}, p = 1, 2$. The estimates are given by

$$\hat{X}_k^1(n) = G_1^1(n) \hat{Z}_k^1(n) + \left(\mu_1^1 - G_1^1(n) \Lambda_1^1\right) \hat{X}_k^1(n),$$

where $\mu_1^1$ is the mean of the estimates $\{\hat{x}_k^1(n)\}_{n=0}^{N-1}, p = 1, 2$. The time-domain estimates $\{\hat{x}_k(n)\}_{n=0}^{N-1}$ are obtained from the frequency-domain estimates $\{\hat{X}_k^1(n)\}_{k=0}^{N-1}$ using the IFFT operation. The mean and variance of the estimates $\{\hat{x}_k(n)\}_{n=0}^{N-1}$ are expressed as

$$\mu_1 = \frac{1}{N} \sum_{k=0}^{N-1} (G_1^1(n) \Lambda_1^1),$$

$$\sigma_1^2 = \mu_1^2 - \nu_1^2 (\mu_1^2),$$

The mean and variance of the second user are obtained in a similar manner to the first user case.

2) Relay-Assisted SC-FDMA with the SIMO Protocol: The receiver structure of relay-assisted SC-FDMA with the SIMO protocol shares the common block of relay-assisted SC-FDMA with the MISO protocol. After SMIC, the received vectors $\mathbf{Z}_1^1$ and $\mathbf{Z}_1^1+1$ of the first user can be expressed in the frequency domain as

$$\mathbf{Z}_1^1 = \sqrt{E_S \mathbf{A}_1^1 \mathbf{X}_1^1} + \sqrt{E_S \mathbf{A}_1^2 \mathbf{X}_2^1} + \mathbf{W}_D^1$$

$$\implies \mathbf{A}_1^1 \mathbf{X}_1^1 + \mathbf{A}_1^2 \mathbf{X}_2^1 + \mathbf{W}_D^1$$

$$\mathbf{Z}_1^1 = \mathbf{A}_1^1 \mathbf{X}_1^1 + \mathbf{A}_1^2 \mathbf{X}_2^1 + \mathbf{W}_D^1,$$ (23)

where $\mathbf{X}_u$ denotes the frequency-domain mean vector from the decoder of the $u$th user. We can change (23) to the matrix form as

$$\mathbf{Z}_1^1 = [\mathbf{Z}_1^1 \ \mathbf{Z}_1^1+1]^T$$

$$= [\mathbf{A}_1^1 \mathbf{X}_1^1 + \mathbf{A}_1^2 (\mathbf{X}_2^1 - \mathbf{X}_2^1)] + [\mathbf{W}_D^1 \ \mathbf{W}_D^1+1],$$

From (24), we derive a linear combining receiver, which is

$$\hat{X}_1^1 = \frac{\Lambda_1^1}{\nu_k^1 |\Lambda_1^1|^2 + \nu_k^2 |\Lambda_1^2|^2 + \sigma_1^2 \nu_k^1},$$

$$= \mathbf{A}_1^1 \mathbf{X}_1^1 + \mathbf{A}_1^2 (\mathbf{X}_2^1 - \mathbf{X}_2^1) + \mathbf{W}_D^1,$$ (25)

where $\hat{X}_1^1$ is the frequency-domain estimate of the first user case. The mean and variance of the estimates $\{\hat{x}_k(n)\}_{n=0}^{N-1}$ are expressed as

$$\mu_1 = \frac{1}{N} \sum_{k=0}^{N-1} (G_1^1(n) \Lambda_1^1),$$

$$\sigma_1^2 = \mu_1^2 - \nu_1^2 (\mu_1^2),$$ (28)
C. EXIT Chart Analysis

The EXIT chart technique is employed to analyze the convergence behavior of the SC-FDMA system with IMD-FDE, by investigating the exchange of mutual information between the IMD-FDE and the decoder. We define $I^E_o = I(L^E_o(c_u(m)); c_u(m))$ and $I^D_o = I(L^D_o(c_u(l)); c_u(l))$ as the output mutual informations from the IMD-FDE and the decoder, respectively. The mutual information is given by [12]

$$I(L; C_u) = \frac{1}{2} \sum_{c_u \in \{+1, -1\}} \int_{-\infty}^{\infty} p(l|c_u) \log_2 \frac{2p(l|c_u)}{p(l+1) + p(l-1)} dl,$$

(29)

where $p(l|c_u)$ denotes the probability density function (pdf) of the LLRs. $p(l|c_u)$ is modelled as outcomes of the random variable $L$ conditioned on its corresponding coded bit $c_u$ of the $u$th user represented by random variable $C_u$. The pdf of LLRs can be estimated from the histogram of the output LLRs. It is shown in [26] that the mutual information can be closely approximated by time average as follows:

$$I(L; C_u) \approx 1 - \frac{1}{N_{\text{blocks}} MN} \sum_{p=1}^{N_{\text{blocks}}} \sum_{m=1}^{MN} \log_2 \left( 1 + e^{-c_{u,p}(m)L(c_{u,p}(m))} \right),$$

(30)

where $c_{u,p}(m)$ is the $m$th coded bit for the $p$th block of the $u$th user, and $L(c_{u,p}(m))$ is the corresponding extrinsic information. This procedure significantly simplifies the computation of $I(L; C_u)$ compared to the histogram method. In this paper, we use the time average approximation method to compute $I^E_o$ and $I^D_o$. For the EXIT chart, the consistent Gaussian approximation is applied to the pdfs of a priori information $L^E_o(c_u(m))$ and $L^D_o(c_u(l))$, i.e., $L_i \sim \mathcal{N}(1/2\sigma_i^2, \sigma_i^2)$. For a chosen $\sigma_i$, the input a priori mutual information $I_i$ is computed numerically by using (29). We can now predict the behavior of the IMD-FDE and the decoder without actually running the algorithm. This is possible by analyzing each constituent component separately via transfer functions. The interleaving and deinterleaving processes allow us to decouple the IMD-FDE and the decoder, which can be analyzed separately. We evaluate the nonlinear transfer function $I_o = \chi(I_i)$, which maps the input a priori mutual information $I_i \in [0,1]$ to the output extrinsic mutual information $I_o \in [0,1]$. Since the extrinsic information generated by the IMD-FDE acts as the a priori information for the decoder and vice versa, we alternately swap the abscissa and ordinate axes in the EXIT chart.

IV. ANALYSIS AND SIMULATION RESULTS

A. The EXIT Chart and BER Performance of IMD-FDE

We consider two users sharing a common set of subcarriers in an SC-FDMA system, over AWGN and multipath fading channels. We assume that the two users have the same power. In the multipath fading environment, each user’s channel is modelled as the six-tap reduced typical urban (TU) channel ($\ell = 25$) [27] with the normalized Doppler frequency $f_D N T_s$ of zero (i.e., fixed channel realization). The length of the information bits and the FFT size are set to $N_b = 65534$ and $N = 256$, respectively. QPSK modulation and a 1/2-rate convolutional code (CC) with generator $G = (7, 5)$ in octal notation are assumed. For the semi-analytic results, at least $10^7$ equi-probable coded bits and their corresponding extrinsic LLRs at the output of the IMD-FDE and the decoder, over several blocks, are used for the computation of extrinsic mutual informations $I^E_o$ and $I^D_o$. Figure 3 shows the transfer characteristics of the IMD-FDE scheme in AWGN and TU channels for various signal-to-noise ratios (SNRs). We can see that for the case of TU channel with SNR = 4dB, the SC-FDMA system with IMD-FDE cannot support two users, because there is an intersection between the EXIT curves of the IMD-FDE and the decoder. It is shown that in AWGN channel, the SC-FDMA system with IMD-FDE provides better mutual information than that in TU channel. However, for perfect a priori information, the EXIT curve of IMD-FDE in TU channel yields almost the same output mutual information.
as in AWGN channel. Figure 3 also shows the trajectory, which is the simulation results taken from the free-running iterative procedure at SNR = 5dB. The system trajectory closely follows the transfer curves, indicating that the EXIT chart analysis is accurate in a given simulation environment.

Figure 4 depicts the simulated BER performance of the SC-FDMA system with IMD-FDE in AWGN and TU channels. As we can see from the EXIT chart analysis, the BER performance in TU channel is degraded for low SNRs. For high SNRs, the SC-FDMA system with IMD-FDE approaches the single-user bound in both AWGN and TU channels. It is shown that the convergence speed of the IMD-FDE in AWGN channel is faster than that in TU channel, which is consistent with the EXIT chart analysis.

B. The EXIT Chart and BER Performance of Relay-Assisted SC-FDMA

We consider the simulation parameters used in Section IV.A. Random interleavers with different interleaving patterns are employed for each user. All underlying links experience frequency-selective channels, where $S_u \rightarrow R$ and $S_u \rightarrow D$, $u = 1, 2$, links are modelled as six-tap TU channels, and $R \rightarrow D$ link is a 2-path channel with a uniform delay power profile. It is assumed that perfect channel estimation is available at the receiver, and the $S_u \rightarrow D$, $u = 1, 2$, and $R \rightarrow D$ links are balanced, i.e., perfect power control. Figure 5 shows the EXIT charts of the relay-assisted SC-FDMA system using IMD-FDE at SNR($S_u \rightarrow R$, $u = 1, 2$) = 20 dB and SNR($S_u \rightarrow D$, $u = 1, 2$) = 4 dB with $f_D N T_s$ of 0.001. In a time-varying and frequency-selective fading channel, the use of the EXIT chart analysis is limited because the pdf of the IMD-FDE output is not a consistent Gaussian distribution [28]. Nevertheless, the EXIT chart can still be used to analyze the performance results, such as the relative performance of various detection schemes, and the impact of certain system parameters on the performance of the iterative process. In Fig. 5, we can observe that the EXIT chart tunnels of the relay-assisted SC-FDMA systems are widely open, whereas there is an intersection between the EXIT curves of the non relay-assisted SC-FDMA and the decoder. For the proposed systems with two users, although the slopes of the curves become steeper, both curves converge to the single-user curves without an intersection. It is also shown that for the first few iterations, the SIMO protocol of the relay-assisted SC-FDMA system provides better mutual information than the MISO protocol of the relay-assisted SC-FDMA system. This is because two consecutive symbol blocks of the other user are viewed as interference terms in the MISO protocol (see (20)), while one symbol block of the other user is viewed as an interference term in the SIMO protocol (see (26)).

Figure 6 compares the BER performance of the non relay-assisted SC-FDMA and two relay-assisted SC-FDMA systems over time-varying and frequency-selective channels at SNR($S_u \rightarrow R$, $u = 1, 2$) = 20 dB with $f_D N T_s$ of 0.001. The single-user performance of each relay-assisted system is included as a lower bound. It is noted that the single-user cases of the proposed MISO and SIMO protocols correspond to the distributed STBC protocol in [24] and the amplify-and-forward protocol in [18], respectively. For fair comparisons, we apply repetition coding to the non relay-assisted SC-FDMA. In the non relay-assisted SC-FDMA, the source repeats its transmission to the destination, so the data rate of non relay-assisted SC-FDMA is the same as those of single-user cases in the relay-assisted SC-FDMA systems. Note that for the non relay-assisted SC-FDMA, a power gain of 3 dB is obtained through repetition. It is shown in Fig. 6 that the relay-assisted SC-FDMA systems achieve cooperative diversity gain and outperform the non relay-assisted SC-FDMA system with repetition protocol. For two users, the proposed relay-assisted SC-FDMA systems with IMD-FDE approach...
the lower bounds after the fifth iteration. This result means that the proposed systems can increase the spectral efficiency of relay-assisted SC-FDMA using a TDMA-based relay protocol. It is noted that the users in the proposed SIMO protocol are silent over the second time slot. Thus, the proposed SIMO protocol is more efficient than the proposed MISO protocol for the non relay-assisted system with repetition protocol, in terms of users’ power consumption [20].

The proposed scheme can be extended to the general case of multiple sources, with low-rate channel coding. Spreading, which can be regarded as a repetition coding, is a simple method to support more than two users [11]. Figures 7(a) and (b) show the EXIT curves of two protocols in the relay-assisted SC-FDMA system with different U (TU channels, $\text{SNR}_{s_u \rightarrow R} = 10\text{dB}$, $\text{SNR}_{s_u \rightarrow D} = 4\text{dB}$, $f_DNT_s = 0.001$, $SF = 2$). (a) MISO protocol. (b) SIMO protocol.

a user-specific interleaver. Since the decoder with $SF = 2$ gives better mutual information than that without spreading, the EXIT chart tunnels are more widely open. It is shown from the EXIT charts that the MISO and SIMO protocols of the relay-assisted SC-FDMA system can support up to five and nine users, respectively. As the number of users increases, the slope of the curve becomes steeper. It means that we need more iterations to converge to the single-user curve. In practical situations, the proposed scheme may need a large number of iterations to support over five and nine users in MISO and SIMO protocols, respectively. Figures 8 shows the BER performance of two protocols with $SF = 2$ for different number of users at $\text{SNR}(S_u \rightarrow R, u = 1, ..., U) = 10\text{dB}$ and $\text{SNR}(S_u \rightarrow D, u = 1, ..., U) = 4\text{dB}$. It is observed that the MISO and SIMO protocols of the relay-assisted SC-FDMA system can support four and six users, respectively, with increased spectral efficiency.

V. CONCLUSIONS

We have presented the IMD-FDE scheme, which can be incorporated into a SC-FDMA framework. The SC-FDMA system with IMD-FDE allows more than one user to share a common set of subcarriers. The EXIT chart analysis was shown to provide insight into the behavior of the IMD-FDE scheme. We have proposed two spectral efficient relay protocols, which convert the spatially distributed antenna systems into effective MISO and SIMO channels. By accessing a relay, destination, or both simultaneously, the proposed protocols increase the spectral efficiency of TDMA-based relay protocol by a factor of two. We have also presented a new iterative multiuser detection scheme, applied to the proposed protocols. Simulation results show that the proposed relay-assisted SC-FDMA with IMD-FDE achieves the diversity gain and approaches the single-user bound in time-varying and frequency-selective fading environments.
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