Bit-Interleaved Coded Multilevel Modulation for Single-Carrier Frequency-Domain Equalization

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Abstract—This paper proposes a bit-interleaved coded multilevel modulation for single-carrier frequency-domain equalization (SC-FDE). In the proposed system, multiple layers are encoded and interleaved together, using one encoder and one interleaver, and then they are mapped by hierarchical symbol mapping. A corresponding receiver structure is presented, where multiple layers can be decoupled by iterative frequency-domain equalization with improved soft information. Simulation results show that the proposed system has better performance than conventional modulation schemes such as bit-interleaved coded modulation (BICM) and multilevel BICM (MLBICM).

Index Terms—Bit-interleaved-coded modulation (BICM), high-order modulation, multilevel modulation, SC-FDE.

I. INTRODUCTION

Single-carrier frequency-domain equalization (SC-FDE) has a similar structure and performance as orthogonal frequency division multiplexing (OFDM) [1]. Recently, the SC-FDE has drawn great attention as an alternative to OFDM, especially in the uplink communications, where lower peak-to-average power ratio (PAPR) benefits the mobile equipment in terms of power efficiency [1]-[3].

High-order modulations such as 16 and 64-QAM can be used to increase the data rate of transmission. For high-order modulations, a minimum mean square error (MMSE) detector with bit-interleaved coded modulation (BICM) has been proposed in [4], [5]. This scheme requires the symbol-to-bit soft demapping in equalization of high-order modulations, resulting in high complexity. Based on multistage decoding, a multilevel coding (MLC) technique was originally proposed by Imai [6]. In [7], multilevel BICM (MLBICM) for single-carrier systems has been proposed. Iterative detection and decoding in MLBICM can detect and decode the multiple coded layers in parallel without the inter-decoder information exchange of multistage decoders, which simplifies the receiver structure. In MLBICM, the symbol-to-bit soft demapping can be avoided due to the linearity of simple constellation mapping. However, since multiple layers have different minimum distances (unequal error probability), the performance of MLBICM is dominated by errors at a layer with a small minimum distance, which degrades its performance.

In this paper, we propose a bit-interleaved coded multilevel modulation for SC-FDE. In the proposed system, multiple layers are encoded and interleaved together, using one encoder and one interleaver, and then they are mapped by hierarchical symbol mapping. Unlike MLBICM, the detector cannot use reliable data decisions due to the single-channel coding. In order to solve this problem, we present an iterative frequency-domain equalization with improved soft information. The equalization combines the equalizer outputs and a priori information to enhance the reliability of estimates of multiple layers. This is related to the partial soft information in [8], [9]. However, we compute this soft information in a new way for the proposed multilevel modulation. Simulation results show that the proposed system outperforms the conventional modulation schemes such as MLBICM and BICM.

II. PROPOSED MULTILEVEL MODULATION

Figure 1(a) depicts the transmitter structure of the proposed system for SC block transmissions. In the multilevel modulation, there are M/2 layers, each of which corresponds to a QPSK sequence. The symbol mapping constructs a 2M-QAM constellation by weighting and adding the QPSK signals. Let $x(x(0) x(1) \cdots x(N-1))^T$ and $X=[X(0) X(1) \cdots X(N-1)]^T$ denote the transmit blocks in the time and frequency domains, respectively. Here, $(\cdot)^T$ is the transpose.

All the information bits are encoded and interleaved together, with one encoder and one interleaver. The interleaved bits are demultiplexed into M/2 multiple layers, $\{c_l(n)\}_{l=0}^{2N-1}$, $l=0, 1, \cdots, M/2 - 1$. Each layer’s sequence is mapped to a QPSK sequence, because each layer transmits the signals over the in-phase and quadrature channels. The QPSK symbol of the lth layer at the nth symbol is obtained as $s_l(n) = (1/2)(c_l(2n) + ic_l(2n + 1))$, $n = 0, 1, \cdots, N-1$, where $i = \sqrt{-1}$, and $c_l(p) \in \{-1, 1\}$, $p = 0, 1, \cdots, 2N-1$.

Hierarchical symbol mapping is employed to construct the $2^M$-QAM constellation. The transmit signal is expressed as

$$x(n) = \sum_{l=0}^{M/2-1} w_l s_l(n), \quad x = (w^T \otimes I_N)s = Ws,$$

where $s = [s_0 s_1 \cdots s_{M/2-1}]^T$, $s_l = [s_l(0) s_l(1) \cdots s_l(N-1)]^T$, $\otimes$ denotes the Kronecker product, and $I_N$ is an $N \times N$ identity matrix. In (1), the mapping vector $w$ is given by

$$w^T = [w_0 w_1 \cdots w_{M-1}] = \begin{cases} [1], & M = 2 \[21]/\sqrt{5}, & M = 4 \[421]/\sqrt{21}, & M = 6. \end{cases}$$
A cyclic prefix (CP) is appended at the head of transmit sequence $x$, and then the block is transmitted over a channel.

At the receiver, after the removal of CP, the received sequence can be expressed as $r = Hx + n = HWS + n$, where $r = [r(0) r(1) \cdots r(N-1)]^T$ and $n$ is a complex additive white Gaussian noise (AWGN) vector with each entry having a zero-mean and variance of $\sigma_n^2$. $H \triangleq \text{Circ}_N[h]$ is an $N \times N$ circulant channel matrix with the first column $h = [h(0) h(1) \cdots h(L-1)]^T$.

### III. Iterative Frequency-Domain Equalization for Proposed Multilevel Modulation

In the single-channel coding scheme which encodes and interleaves all the layers in a block, decoding cannot be done until all the layers are processed. Thus, the equalization cannot use reliable data decisions, and the residual interference between the layers degrades its performance. To overcome this problem, we derive the iterative frequency-domain equalization with improved soft information for the proposed system.

Figure 1(b) shows the receiver block diagram of the proposed system. The detection order of multiple layers is determined to adopt the successive detection. The layer with the largest minimum distance $(\nu_0)$ is detected first. At the $l$th detection stage, the extrinsic information of layers, $s_j(n)$, $j = 0, 1, \cdots, l-1$, has already been computed by the equalizer. Thus, the extrinsic information is fed back and added to the soft output channel decoder’s extrinsic information to enhance the estimation of transmit symbols. The improved soft information can be computed as follows

$$L_F(c_j(p)) = L_F^e(c_j(p)) + L_F^e(c_j(p)), \quad p = 0, 1, \cdots, 2N-1, \quad j = 0, 1, \cdots, l-1,$$

where $L_F^e(c_j(p))$ and $L_F^e(c_j(p))$ are the a priori and extrinsic LLR computed by the decoder and equalizer, respectively. Here, the superscript $E$ of $L_F^e$ represents the equalizer. The subscripts $I$ and $O$ represent the input and output of the equalizer, respectively. To perform MMSE equalization, the mean and variance of coded symbols $\{s_j(n)\}_{n=0}^{N-1}$ are required, which are obtained as

for $l \leq j < M/2$,

$$\hat{s}_j(n) = E[s_j(n)\mid L_F^e(c_j(p))] = \frac{2}{\sqrt{2}} \left( \frac{\text{tanh} \left( \frac{L_F^e(c_j(2n))}{2} \right) + \text{tanh} \left( \frac{L_F^e(c_j(2n+1))}{2} \right)}{2} \right),$$

and for $0 \leq j < l$,

$$\hat{s}_j(n) = E[s_j(n)\mid L_F(c_j(p))] = \frac{2}{\sqrt{2}} \left( \frac{\text{tanh} \left( \frac{L_F(c_j(2n))}{2} \right) + \text{tanh} \left( \frac{L_F(c_j(2n+1))}{2} \right)}{2} \right),$$

where $(\cdot)^H$ denotes the conjugate transpose. Under the MMSE criterion, the $N \times 1$ equalizer vector $g_n^e$ is given by

$$g_n^e = \text{Conv}(r, r)^{-1} \text{Cov}(r, s_l(n)) = [HVVW^H H^H + \sigma_n^2 I_N]^{-1} \hat{v}_l(n)w_l H e_n,$$

where $V = \text{Diag}[v_0 v_1 \cdots v_{N/2-1}]$ represents the covariance matrix of $s_l$ and $e_n = [\delta_{1,n} 0 1 0_{1 \times (N-n-1)}]^T$. $V_j = \text{Diag}[v_j(0) v_j(1) \cdots v_j(N-1)]$, and the $(n, n)$ entry of covariance matrix $V_j$ is given by

$$v_j(n) \triangleq \left\{ \begin{array}{ll} \hat{v}_j(n), & j = 0, 1, \cdots, l-1 \\ \hat{v}_j(n), & j = l, l+1, \cdots, M/2-1. \end{array} \right.$$
Gaussian distribution, the extrinsic LLRs, \( L_E^c(c(p)) \), \( p = 0, 1, \cdots, 2N - 1 \), can be obtained as
\[
L_E^c(c(2n)) = \frac{\sqrt{SRe\{\hat{s}_l(n)}\}}{1 - v_l \mu_l}, \quad L_E^c(c(2n+1)) = \frac{\sqrt{SIm\{\hat{s}_l(n)}\}}{1 - v_l \mu_l}.
\]

The extrinsic LLRs, \( \{L_E^c(c(p))\}_{p=0}^{2n-1} \), are used in (3) for the \( l+1 \)th layer’s detection. After all the layers are detected by the equalizer, the extrinsic LLRs of all the layers are deinterleaved and fed to the MAP decoder. The MAP decoder computes the extrinsic information for the coded bits, which is used as the a priori information \( L_E^c(c(p)) \) in (3) and (4) for ILI cancellation and per-tone MMSE equalization.

IV. SIMULATION RESULTS

We consider a coded SC block transmission with \( N=256 \), and 16-QAM constellation. A 1/2-rate convolutional code with a constraint length of 3 is employed, and random interleavers are used. The CP length is set to the channel maximum delay. For a time varying and frequency-selective fading channel, a 6-tap reduced typical urban (TU) channel is considered. In the simulations, BICM uses the standard Gray-mapped 16-QAM. For fair comparisons, the iterative equalizers presented in [4] and [7] are applied to BICM and MLBICM, respectively. The number of iterations is 5.

Figure 2 shows the block error rate (BLER) performance of proposed system with or without the improved soft information. The proposed system without the improved soft information detects the layers in parallel. We observe from Fig. 2 that the improved soft information significantly enhances the performance of the proposed system, due to the improvement in the reliability of the estimates. In Fig. 2, the proposed system is also compared to MLBICM and BICM. It is shown that the proposed system outperforms BICM. MLBICM has unequal error protection capabilities because of the difference in minimum distance between layers [7]. The BLER performance of MLBICM is dominated by errors at the second layer. In contrast, the proposed system averages the performance of multiple layers with different minimum distances. As such, the proposed system provides better performance than MLBICM. Figure 3 shows the BLER performance of BICM, MLBICM, and the proposed system for different iteration numbers. Before the second iteration, the proposed system shows the worst performance, due to the residual interference between the layers. However, by employing the iterative equalization with the improved soft information, the interference diminishes as the number of iterations increases, and the proposed system achieves a coding gain. In the simulations, we assumed perfect channel state information at the receiver. Highly accurate channel estimation can be done using the frequency-domain multiplexed pilot technique proposed in [11].

V. CONCLUSIONS

This paper proposed a bit-interleaved coded multilevel modulation for SC-FDE. In order to average the performance of multiple layers with different minimum distances, the proposed system encodes and interleaves multiple layers together, using one encoder and one interleaver. A corresponding receiver structure was presented to decouple the multiple layers which are encoded and interleaved together. Simulation results show that the proposed system achieves a coding gain by using single-channel coding, and outperforms the conventional modulation schemes.

REFERENCES