Decision-Directed Channel Estimation for SC-FDE in Amplify-and-Forward Relaying Networks

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Abstract—In this letter, we propose a decision-directed channel estimation scheme for single-carrier frequency-domain equalization in amplify-and-forward (AF) relaying networks. For coherent combining of the received signals in AF-based cooperation, conventional channel estimation schemes have focused on estimation of source-destination and source-relay-destination channels. However, if the individual channels in the source-relay-destination link are not considered, the relaying gain is limited because the noise propagation from the relay to the destination affects the effective signal-to-noise ratio at the destination. The proposed scheme estimates the effective channel parameters for maximal-ratio combining, in the absence of channel-state information (CSI) from the relay. Simulation results show that the proposed scheme outperforms the conventional scheme and approaches the performance with perfect CSI.

Index Terms—Single-carrier frequency-domain equalization (SC-FDE), decision-directed channel estimation (DDCE), amplify-and-forward (AF) relay.

I. INTRODUCTION

**SC**INGLE-CARRIER with frequency domain equalization (SC-FDE) has similar performance and essentially the same overall complexity as orthogonal frequency division multiplexing (OFDM) [1]. Recently the SC-FDE has drawn great attention as an alternative to the OFDM, especially in the uplink communications, where lower peak-to-average power ratio significantly increases the transmit power efficiency of mobile terminals.

Spatial diversity is a powerful technique to combat the fading effect in wireless communications, through the use of multiple antennas at the transmitter and/or receiver sides. Unfortunately, the use of multiple antennas might be impractical in mobile devices due to the limitation of size and complexity. Cooperative diversity overcomes these problems without the additional complexity of multiple antennas [2], [3]. Multiple terminals in the network cooperate to form a virtual antenna array realizing spatial diversity in a distributed fashion. Amplify-and-forward (AF) or decode-and-forward (DF) schemes are basic cooperative protocols, which are generally considered in communication networks. Compared to DF scheme, AF relaying can achieve diversity gain without special strategies and has less processing burden on the relays [2].

For coherent combining of the signals from source-destination ($S \rightarrow D$) and source-relay-destination ($S \rightarrow R \rightarrow D$) channels, both channels should be estimated. For the $S \rightarrow D$ channel, conventional channel estimation schemes can be applied, and have been well addressed for SC-FDE systems [4], [5]. In an AF relay channel ($S \rightarrow R \rightarrow D$), the effective channel which consists of the source-relay ($S \rightarrow R$) and the relay-destination ($R \rightarrow D$) links is non-Gaussian. Moreover, because of the noise propagation from the relay to the destination, the effective noise caused by the relay and destination is not additive white Gaussian noise (AWGN). In [6], the authors proposed the linear minimum mean-square error (LMMSE) estimator for $S \rightarrow R \rightarrow D$ channel, assuming that the variance of the effective noise is available at the destination. In practice, however, the variance of the effective noise needs to be estimated for maximal-ratio combining in AF-based cooperative systems. This variance can be calculated by exploiting the information of the $S \rightarrow R$ channel in the disintegrated channel estimation (D-CE) scheme [7] which separately estimates the $S \rightarrow R$ and $R \rightarrow D$ channels. However, the D-CE scheme requires a channel estimator at the relay and increases signaling overhead by feed-forwarding the $S \rightarrow R$ channel estimates to the destination.

In this letter, we propose a decision-directed channel estimation (DDCE) scheme for SC-FDE in AF relaying networks. The proposed scheme estimates the effective channel parameters in the $S \rightarrow R \rightarrow D$ link for maximal-ratio combining, without the channel-state information (CSI) from the relay. By considering the effective noise in the detection process, the proposed scheme prevents the error floor caused by noise propagation, and gives better performance than the conventional scheme.

II. SYSTEM MODEL

We consider a relay-assisted SC block transmission over time- and frequency-selective channels. The channel impulse response (CIR) for transmitting node $A$ to receiving node $B$ is given by $h_{AB}(m) = [h_{AB}(m,0),\ldots,h_{AB}(m,L_{AB})]^T$, where $m$ is the block index in the frame, and $L_{AB}$ is the CIR length. Subscripts $S$, $R$, and $D$ stand for source, relay, and destination nodes, respectively. A frame is composed of $M$ modulated information blocks, i.e., $\{x_{S}(m)\}_{m=0}^{M-1}$. Each block is described as $x_{S}(m) = [x(m,0),\ldots,x(m,N-1)]^T$, where $N$ is the block length. During the first slot, the source transmits the frame after appending a cyclic prefix (CP) with length $L_{CP} \geq \max(L_{SD},L_{SR} + L_{RD})$ in the head of each block. The CP eliminates inter-block interference and makes the channel matrix circulant, i.e., $\mathbf{H}_{AB}(m)_{i,j} = h_{AB}(m,(i-j) \mod N)$. The received signals at the relay and the destination are written as

\[
\begin{align*}
    y_{R}(m) &= \sqrt{E_{SR}}\mathbf{H}_{SR}(m)x_{S}(m) + w_{R}(m), \\
    y_{D1}(m) &= \sqrt{E_{SD}}\mathbf{H}_{SD}(m)x_{S}(m) + w_{D1}(m),
\end{align*}
\]

\[
m = 0,1,\ldots,M-1
\]
where \( w_R(m) \) and \( w_{D1}(m) \) are AWGN vectors with each entry having zero mean and a variance of \( N_0/2 \) per dimension. \( E_{AB} \) represents the average energy available at the receiving node \( B \), and includes path loss and shadowing effects in the \( A \rightarrow B \) link. To guarantee unit average energy, the relay normalizes the received signal as \( x_R(m) = \alpha y_R(m) \) where the scaling factor \( \alpha = 1/\sqrt{E_{SR} + N_0} \).

During the second slot, the relay transmits the normalized signal to the destination. The received signal at the destination is expressed as

\[
y_{D2}(m) = \sqrt{E_{RD}} H_{RD}(m)x_R(m) + w_{D2}(m)
\]

\[
H_{SRD}(m) \triangleq \alpha \sqrt{E_{SRD}} H_{SR}(m),
\]

\[
w_{SRD}(m) \triangleq \alpha \sqrt{E_{SRD}} w_R(m) + w_{D2}(m).
\]

Since circular matrices are transformed to diagonal matrices by the discrete Fourier transform matrix \( F \), the \( k \)th frequency responses of received signals in the \( m \)th block are represented by

\[
Y_{D1}(m, k) = \Lambda_{SD}(m, k) X_S(m, k) + W_{D1}(m, k),
\]

\[
Y_{D2}(m, k) = \Lambda_{SRD}(m, k) X_S(m, k) + W_{SRD}(m, k),
\]

\[
0 \leq m \leq M - 1, \quad 0 \leq k \leq N - 1.
\]

\( \Lambda_{SD}(m, k) \) and \( \Lambda_{SRD}(m, k) \) are channel frequency responses of the \( S \rightarrow D \) and \( S \rightarrow R \rightarrow D \) links, respectively. We define them as

\[
\Lambda_{SD}(m, k) \triangleq \sqrt{E_{SD}} \left[ F H_{SD}(m) F^\dagger \right]_{k,k},
\]

\[
\Lambda_{SRD}(m, k) \triangleq \sqrt{E_{SRD}} \left[ F H_{SRD}(m) F^\dagger \right]_{k,k},
\]

where \((\cdot)^\dagger\) denotes the complex-conjugate transpose. The noise components in the frequency domain \( W_{D1}(m, k) \) and \( W_{SRD}(m, k) \) are given by \([F w_{D1}(m)]_k\) and \([F w_{SRD}(m)]_k\), respectively.

### III. PROPOSED CHANNEL ESTIMATION SCHEME IN AF RELAYING NETWORKS

LMMSE channel estimation is optimum in the sense of minimizing MSE when the receiver knows the channel statistics. However, a channel estimator which tightly matches the channel statistics is difficult to design, because the statistical information may not be available at the receiver. Robust LMMSE channel estimation [8] does not require a priori knowledge of channel correlation by assuming a uniform power delay profile and a Doppler power spectrum. The performance degradation caused by correlation mismatch can be overcome by using a decision-directed approach in which the detected data tones are exploited as pilots for channel estimation [5]. Figure 1 is the block diagram of the proposed channel estimation scheme, whose procedure is described as follows.

1) **Slot 1**: Using the least square (LS) method, the channel estimates are obtained at the pilot positions defined as a set \( \Phi_P \).

\[
\hat{\Lambda}_{SD}^{(0)}(m, k) = \frac{Y_{D1}(m, k)}{X_S(m, k)}, \quad (m, k) \in \Phi_P
\]

To reduce the receiver complexity, we apply linear interpolation (LI) and obtain \( \hat{\Lambda}_{SD}^{(0)}(m, k) \) for all subcarriers in the frame. \( \hat{\Lambda}_{SD}^{(0)}(m, k) \) is then used by the MMSE equalizer, producing the tentative decision of transmitted data \( \hat{b}(m, n) \). These data are converted to the frequency domain symbol \( \hat{X}_S(m, k) \), which is modeled as a complex Gaussian random variable. \( \hat{X}_S(m, k) \) with low power generates a large MSE on LS estimates. By using frequency replacement (FR), these estimates are replaced with the channel estimates used in the previous iteration, i.e.,

\[
\hat{\Lambda}_{SD}^{(1)}(m, k) = \begin{cases}
\frac{Y_{D1}(m, k)}{X_S(m, k)}, & |\hat{X}_S(m, k)| > \lambda,
\hat{\Lambda}_{SD}^{(0)}(m, k), & \text{otherwise},
\end{cases}
\]

where \( \lambda \) is the threshold value [5]. The updated estimate \( \hat{\Lambda}_{SD}^{(1)}(m, k) \) produces the final estimate \( \hat{\Lambda}_{SD}^{(1)}(m, k) \) through a cascaded 2×1D FIR Wiener filter (WF).

2) **Slot 2**: As in the slot 1, \( \hat{\Lambda}_{SRD}^{(0)}(m, k) \) is acquired for the whole subcarriers in the frame. By linear combining, the combined signal \( Y_D(m, k) \) is obtained as

\[
Y_D(m, k) = [G_1(m, k) G_2(m, k)] \begin{bmatrix} Y_{D1}(m, k) \\ Y_{D2}(m, k) \end{bmatrix}
\]

where the weighting coefficients are given by

\[
G_1(m, k) = \frac{\hat{\Lambda}_{SD}^{(1)}(m, k)}{N_0}, \quad G_2(m, k) = \frac{\hat{\Lambda}_{SRD}^{(0)}(m, k)}{N_0},
\]

and \((\cdot)^*\) denotes the complex-conjugate. Since the CSIs of the \( S \rightarrow R \) and \( R \rightarrow D \) links are not available, we cannot calculate the exact variance of the effective noise, and thus we approximate it to \( N_0 \). Then, we perform MMSE equalization. Equalizer tap coefficients \([C(m, k)]_{k=0}^{N-1}\) are derived as

\[
C(m, k) = \frac{1}{\hat{\Lambda}(m, k) + 1},
\]

where the estimate for equivalent channel \( \hat{\Lambda}(m, k) \) is expressed as

\[
\hat{\Lambda}(m, k) = G_1(m, k) \hat{\Lambda}_{SD}^{(1)}(m, k) + G_2(m, k) \hat{\Lambda}_{SRD}^{(0)}(m, k).
\]

After equalization, \( \hat{X}_S(m, k) \) is obtained and used for FR. The output of FR, \( \hat{\Lambda}_{SRD}^{(1)}(m, k) \), produces the final estimates \( \hat{\Lambda}_{SRD}^{(1)}(m, k) \) using the same WF as in the slot 1.

Inaccurate variance of the effective noise may generate error floor because the propagated noise is not considered in the detection process. Thus, we estimate the variance as an effective channel parameter by using the effective noise level measurement (ENLM) technique. The distribution of \( W_{SRD}(m, k) \) is neither white nor Gaussian. There exists a distinct \( E[|W_{SRD}(m, k)|^2] \) corresponding to each \((m, k)\), but it is hard to calculate the ensemble average of the noise power for every subcarrier. Instead, we measure time average of the noise power in the frame. From (4),
The measured $\sigma^2_{SRD}$ is used for linear combining. In (9), we update $G_2$ to $\tilde{G}_2$, where $\tilde{G}_2(m,k) = \tilde{\Lambda}^{(1)}_{SRD}(m,k) / \sigma^2_{SRD}$. We can also find the updated equivalent channel by substituting $\tilde{G}_2(m,k) = \tilde{\Lambda}^{(1)}_{SRD}(m,k)$ for $G_2(m,k) = \Lambda^{(1)}_{SRD}(m,k)$ in (10). After MMSE equalization, the final decision of the transmitted data is produced.

IV. SIMULATION RESULTS

We investigate the frame error rate (FER) performance of the proposed channel estimation scheme in AF relaying networks. In the simulation, each frame consists of six data blocks and one reference block located in the middle of the frame (i.e., $M=7$) [9]. Each data block has 512 QPSK symbols ($N=512$). In reference block, 128 of 512 symbols are used for channel estimation ($P=128$). As a smoothing filter, we employ WF which uses the nearest 17 symbols in the frequency domain. FR threshold $\lambda$ is 0.3 [5]. All underlying links are modelled as ITU vehicular A (VA) channels [10], and the S→D and R→D links are balanced. Transmission bandwidth is 8 MHz and the normalized Doppler frequency is 0.001.

Figure 2 shows the performance of the proposed scheme when $E_{SR}/N_0$ is fixed to 15 dB. The performance of the repetition protocol is included to show the gain of AF relaying. The conventional channel estimation scheme in [5] uses two different WFs for interpolation and smoothing, which is denoted as ‘WF - WF’ in Fig. 2. When this scheme is applied to the AF relaying, the error floor is observed because the propagated noise is not considered in the detection process. The proposed scheme, denoted as ‘LI - WF with ENLM’, provides better performance than the conventional scheme, and its performance approaches the lower bound with 0.7 dB gap.

The overall computational complexity of LI and ENLM used for the proposed scheme is $O(MN)$, whereas WF used for interpolation in the conventional scheme has computational complexity of $O(P^2N)$. Figure 3 shows the performance of the proposed DDCE scheme for different S→R link qualities. $E_{SR}/N_0$ and $E_{RD}/N_0$ are fixed to 21 dB. It is shown that the proposed scheme outperforms the conventional scheme, especially when $E_{SR}/N_0$ is low.

V. CONCLUSIONS

A new DDCE scheme of SC-FDE has been proposed for AF relaying networks. The proposed scheme estimates the effective channel parameters for maximal-ratio combining, without side information from the relay. Simulation results show that the proposed scheme outperforms the conventional scheme and approaches the performance with perfect CSI.

REFERENCES